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THE

WESTERN CALCULATOR,

OR A

NEW AND COMPENDIOUS SYSTEM

OF

PRACTICAL ARITHMETIC;

CONTAINING

THE ELEMENTARY PRINCIPLES AND RULES OF CALCULATION IN WHOLE, MIXED, AND DECIMAL NUMBERS,

ARRANGED, DEFINED, AND ILLUSTRATED,

IN A PLAIN AND NATURAL ORDER;

ADAPTED TO THE USE OF SCHOOLS, THROUGHOUT THE WESTERN COUNTRY AND PRESENT COMMERCE OF THE UNITED STATES.

IN EIGHT PARTS.

BY J. STOCKTON, A. M.

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PREFACE.

Among the many systems of Arithmetic now used in our American schools, though each has its individual merit, yet all contain many things which are either entirely useless, or of but little value to most beginners.

It is to be regretted also, that in most of these systems, even in those parts which are valuable and important, the authors appear not to have been sufficiently aware of giving a plain and natural arrangement and system to the whole. There is not that visible connexion between the parts, which enables the attentive pupil to discover, as he progresses, that he is learning a system, and not a number of separate and unconnected rules.

In many things, also, more attention has been given to gratify the inquiries of the *proficient*, than to furnish plain, but necessary instruction to the *beginner*. The age, capacity, and progress of the scholar are also overlooked; and a mode of instruction too learned, and too elaborate, is pursued. It is forgotten how difficult even the most simple parts are to a young mind; nor are the instances few, in which even the variety of ways laid down, in which the same question may be solved, leaves the learner perplexed, and swells the size of the work.

To remedy, in some measure, these defects, and to furnish our numerous schools, in the western country, with a plain and practical treatise of Arithmetic, compiled and printed among ourselves, thereby saving a heavy annual expense in the purchase of such books, east of the mountains, and likewise the carriage thereof, have been the motives which induced the compiler to undertake this work.

In it the following objects have been steadily kept in view:

1st. Plainness and simplicity of style, so that nothing should be introduced above the common capacities of scholars, at the early age in which they are generally put to the study of Arithmetic.

2d. A natural and lucid arrangement of the whole, as a system, in which the connexion and dependence of all the parts may be easily discovered and understood. To accomplish this object, the work is divided into eight parts, following each other, in what appears to the compiler the natural and simple divisions of the science. Each of these parts is again divided into sections, following the same connected arrangement. In each of these sections, the rules are expressed in a

short and plain manner, and each rule is illustrated, with a few easy and familiar examples, gradually proceeding from that which is simple, to such as are more abstruse and difficult.

3d. Clearness and precision in the definitions, directions, and examples. Carefully explaining every technical term when first used, and thereby guarding against ambiguity and uncertainty.

4th. Brevity in each part, so that every thing useless, or unimportant, may be excluded; in order that the work may find its way into schools at the cheapest rate; that parents, when examining the schoolbooks of their children, may not find, whilst one part is worn out, the other is untouched, and half the price of the book entirely lost.

How far these objects have been obtained, must be left to the decision of time. Should the work be found to aid the progress of scholars, in acquiring a practical knowledge of this useful science—to save expenses in a book, so many of which are required; and be found a useful assistant to merchants, mechanics, and farmers, as well as in some degree to lessen the labor of teachers in this branch of the sciences; the compiler will have obtained his object.

NOTICE TO THE FOURTH EDITION.

The favorable reception, and wide circulation, of the former editions of the *Western Calculator*, stimulate the author to make it still more deserving of public patronage.

He has, therefore, at the suggestion of several respectable teachers, given sundry additional questions to some of the rules; and also some other alterations, which several years' experience in teaching has pointed out.

The greatest care has been taken to prevent errors from appearing in this edition.

Pittsburgh, February 1, 1823.

APHORISMS

FOR THE

SCHOLAR'S CAREFUL CONSIDERATION AND ATTENTION.

KNOWLEDGE is the chief distinction between wise men and fools; between the philosopher and the savage.

The common and necessary transactions of business cannot be conducted with profit or honesty, without the knowledge of Arithmetic.

He who is ignorant of this science must often be the dupe of knaves, and pay dear for his ignorance.

Banish from your mind, idleness and sloth, frivolity and trifling; they are the great enemies of improvement.

Make study your inclination and delight; set your hearts upon knowledge.

Accustom your mind to investigation and reflection; determine to understand every thing as you go along.

Commit every rule accurately to memory, and never rest satisfied until you can apply it.

As much as possible do every thing yourself; *one* thing found out by your own study, will be of more real use than *twenty* told you by your teacher.

Be not discouraged by *seeming* difficulties; patience and application will make them plain.

Endeavor to be always the best scholar in your class, and to have the fewest mistakes, or blots, in your book.

"The wise shall inherit honor, but shame shall be the promotion of fools."

EXPLANATION

OF THE

SEVERAL CHARACTERS EMPLOYED FOR THE SAKE OF BREVITY, IN THIS TREATISE.

=Two parallel lines, signifying equality: as, 100 cents= 1 dollar; that is, 100 cents are equal to 1 dollar.

+Signifying *more*, or addition: as, 6+4=10; that is, 6 and 4 added make 10. This character is called ***Plus.***

—A single line, signifying *less*, or subtraction: as, 6—4= 2; that is, 6 less 4 is equal to two. This character is called ***Minus.***

× Signifying ***Multiplication:*** as, 2×4=8; that is, 2 multiplied by 4 is equal to 8.

÷Signifying ***Division:*** as, 6÷3=2; that is, 6 divided by 3 is equal to 2.

:: :Signifying ***Proportion:*** as, 2 : 4 :: 6 : 12; that is, as 2 is to 4, so is 6 to 12; or, that there is the same proportion between 6 and 12, as there is between 2 and 4.

$\surd$ or $\surd$ Signifying the square root of the number before which it is placed: as, $\surd 64=8$; that is, the square root of 64 is 8.

$\sqrt[3]{}$ Signifying the cube root: as, $\sqrt[3]{64}=4$; that is, the cube root of 64 is 4.

———A Vinculum, or chain: denoting the several quantities over which it is placed, are to be considered as one simple quantity

THE

WESTERN CALCULATOR.

PART I.

ARITHMETIC IN WHOLE NUMBERS.

ARITHMETIC is the art, or science, of computing by numbers, and is generally divided into five principal parts, or primary rules: *viz.* Numeration, Addition, Subtraction, Multiplication, and Division.

SECTION 1.

OF NUMERATION.

NUMERATION (or, as it is often called, *Notation*) is the art of expressing any given or supposed number, by the ten following characters: 0, 1, 2, 3, 4, 5, 6, 7, 8, 9. The first of these is called a cipher, the rest are termed digits or figures.

These nine digits are divided into three periods, three in each period. The first period includes units, tens, and hundreds. The second period includes thousands, tens of thousands, hundreds of thousands. The third period includes millions, tens of millions, hundreds of millions.

Note.—The cipher is also called *nought* and *zero*. They are Arabic characters.

The relative value of each period, and the different figures in each period, may be learned from the following

TABLE.

3d period.			2d period.			1st period.		
Hund. of Millions	Tens of Millions	Millions	Hund. of Thousands	Tens of Thousands	Thousands	Hundreds	Tens	Units
9								
9	8							
9	8	7,						
9	8	7,	6					
9	8	7,	6	5				
9	8	7,	6	5	4,			
9	8	7,	6	5	4,	3		
9	8	7,	6	5	4,	3	2	
9	8	7,	6	5	4,	3	2	1,

After the foregoing table, with the preceding definitions and explanations, are well explained by the teacher, and accurately committed to memory by the pupil, let him next proceed

To write Numbers,

Observing carefully the following

RULE.

Write down first, the given sum, in such figures as express its value, and then supply the deficiencies therein with ciphers

Application.

Write down in figures the following numbers.

1. Sixteen.
2. Forty-nine.

3. Three hundred and eighty-five.
4. Two thousand six hundred and ten.
5. Sixty-four thousand, five hundred and thirty-six.
6. Two hundred and fifty-three thousand, eight hundred and forty-two.
7. Five millions, six hundred thousand and six.
8. Ninety millions, three hundred and five.
9. Eight hundred and twenty-nine millions, six thousand and two.

Write down in words at length the following sums:

5, 17, 35, 458, 6829, 72348, 384721, 2683200, 50678024.

Numbers are also expressed by letters, and are called *numeral letters*, or Roman numbers. Thus,

1	2	3	4	5	6	7	8	9	10	11	20	30
I,	II,	III,	IV,	V,	VI,	VII,	VIII,	IX,	X,	XI,	XX,	XXX,

40	41	50	60	70	80	90	100	200	500	1000
XL,	XLI,	L,	LX,	LXX,	LXXX,	XC,	C,	CC,	D,	M.

When a letter of less value stands before one of a greater, it diminishes, but when placed after, it increases, the value of the greater.

SECTION 2.

OF ADDITION.

ADDITION is of two kinds, *viz.* Simple and compound.

Simple addition teacheth to collect two or more numbers of the same denomination into one sum: as, 6 dollars and 4 dollars, make 10 dollars.

RULE 1. Write the different numbers in the given sum in such a manner, that the units may stand under units, the tens under tens, the hundreds under hundreds, &c. till the whole is set down.

2. Begin with the column of units, and add it into one sum, carrying to the next column, one for every ten, and set down the remainder directly underneath: proceed in the same manner from tens to hundreds, &c. till all is finished.

Prove the work by reckoning downwards as well as upwards, and if the amounts be equal the work is right.

EXAMPLES.

Dollars		Yards		Feet
2465	2.	468256	3.	647502434
4832		348928		961948260
6143		764182		959276398
13440 added upwards.				
13440 added downwards.				

4.	258335091	5.	237680923	6.	919283746
	138097653		423315687		213536978
	573217809		098172635		321325687
	532458976		523516533		978562313
	532175633		653213563		321897553
	249753290		327865309		213587921
7.	5643218624	8.	4	9.	6856789436
	135940536		45		40590428
	42006302		456		36491
	9580469		4567		2849653210
	550214		45678		540
	32651		456789		34906
	4168		4567890		8458000
	324		45678901		300
	68		456789012		9
	4		4567890123		35

10.	3683678048934	11.	595536210486
	2864948946496		376891345613
	8498649476828		765248567416
	3646280568245		684720476828
	6421424678427		852134567812
	3678156496862		236889634567
	7654584964859		335678902345

EXAMPLES.

Method 1.		Method 2.	
246	425	425	2 } 6
425	246	246	3 }
1230	2550	2550	
492	1700	1700	
984	850	850	
104550	104550	104550	6

Note. This last method is not *absolutely* certain; yet the probability is so great, that in general it may be relied on.

3. Multiply 5221 by 145 *Ans.* 757045
4. 23430 230 5388900
5. 3800920 80750 306924290000
6. 89536925 735 65809639875
7. 78965987 5893 465346561391

8. What will 75 bushels of wheat come to at 1,15 cents per bushel? *Ans.* 86 dolls. 25 cents.

9. Bought 3950 lbs. of coffee, at 29 cts. per lb. what must I pay? *Ans.* 1145 dolls. 50 cents.

10. There are 12 pence in one shilling. How many are there in 40? *Ans.* 480 pence.

Case 3.

When the multiplier is the exact product of any two factors in the multiplication table.

RULE.

Multiply the given sum by one of these; and that product multiplied by the other, will give the number required.

EXAMPLE.

1. Multiply 4236 by 16.

4
16944
4
Product 67776 } 4×4=16.

2. Multiply 871075 by 21 *Ans.* 18292575
3. 2453642 36 88331112
4. 43102 64 2758528
5. 23645 144 3404880
6. 12071 99 1195029

Case 4.

When there are ciphers at the right of one or both the factors.

RULE.

Omit them in the operation, but annex them to the product

EXAMPLE.

1. Multiply 240 by 20.

24.0
2.0

48.00 *Ans.* 4800

2.	3600	by	400	1440000
3.	44000		550000	24200000000
4.	663000		60000	39780000000

Note. When the multiplier is 10, the product will be found by adding one cipher to the multiplicand; if 100, add two ciphers; if 1000 add three; &c.

EXAMPLE.

1. Multiply	200	by	10	*Ans.* 2000
2.	462		100	46200
3.	879		1000	879000

Application.

1. A gentleman owes 25 laborers 15 dollars each; how much does the whole come to? *Ans.* 375 dolls.

2. A saddler owes his journeyman for 43 days' work, at 125 cents per day; how much does he owe him in all?

Ans. 53 dolls. 75 cts.

3. A merchant buys 440 yards of muslin at 32 cents per yard; how much does the whole cost? *Ans.* 140 dolls. 80 c.

4. A farmer sells 60 bushels of wheat at 125 cents per bushel; 40 bushels of rye at 85 cents; 34 of corn at 50 cents; how much is he to receive for each, and how much does the whole amount to?

Ans. 75,00 cents for the wheat, 34,00 cents for the rye 17,00 cents for the corn; and the whole amounts to 126,00 cents, or 126 dollars.

5. A dollar is equal to 10 dimes, and a dime is equal to 10 cents; how many dimes and cents are there in 100 dollars? *Ans.* 1000 dimes, and 10,000 cents.

6. How many panes of glass are there in a house that has 32 windows, 20 of which have 24 lights each, and the *have 18 each*? *Ans.* 696 panes.

7. What sum is equal to 7525 multiplied by 125?

Ans. 940625.

8. A has 250 dollars, B has three times as many, and C has four times as many as B; how many dollars have B and C each, and how many have they altogether?

Ans. B has 750 dolls. C. 3000 dolls. altogether $4000.

SECTION 5.

OF DIVISION.

Division is either simple or compound.

Simple division is finding how often one number is contained in another of the same name, or denomination.

The number given to be divided, is called the *dividend.*

The number given to divide by, is called the *divisor.*

The result, or answer, is called the *quotient.*

Case 1.

When the divisor does not exceed 12.

RULE.

Find how often the divisor is contained in the first figure or figures in the dividend, under which set the result, if any remain, conceive it as so many tens added to the next figure, and then proceed in the same manner.

Division is proved by multiplying the quotient by the divisor, and adding the remainder, if any: the amount will equal the dividend.

EXAMPLES.

Divisor	2)46578238	3)672245139	4)4756394344
Quotient	23289119	224081713	
	2	3	
Proof	46578238	672245139	
	5)97036142	8)37846210	12)64381259
	6)3824966	7)46825486	9)8297463813

Case 2.

When the divisor exceeds 12.

RULE.

Begin with as many of the first figures in the dividend as will contain the divisor. Try how often the divisor is contained therein, and set the result in the quotient.—Subtract the product of the divisor multiplied by the quotient figure from the dividual above, to this remainder annex the next figure in the dividend for a new dividual, and so proceed till all the figures in the dividend are brought down.

Note. A *dividual* is when one or more figures of the dividend, (in the operation of long division) are divided separately from the rest.

EXAMPLES.

	Dividend.		Quot.			
Divis. 42)	9870	(235		41)94979	(2316	
	84	42		82	41	
	147	470		129	2316	
	126	940		123	9264	
	210	9870	proof.	67	94956	
	210			41	23	rem.
				269	94979	pr.
				246		
				23 Rem.		

			Quotient.		
3.	Divide 29687624	by 64	*Ans.* 463869	and 8	Rem.
4.	47989536925	735	65291886	715	
5.	4917968967	2359	2084768	1255	
6.	5374608	671	8009	569	
7.	19842712000	175296	113195	81280	
8.	5704392	108	52818	43	

Case 3.

When the divisor is the exact amount of any two factors in the table.

RULE.

Divide the given sum by any one of these, and the quotient by the other.

EXAMPLE.

Divide 9870 by 42.

6)9870

7)1645

235 *Ans.*

Case 4.

When one or more ciphers stand on the right of the divisor.

RULE.

Omit them in the operation, cutting off from the right of the dividend as many figures, taking care to annex them to the remainder.

EXAMPLE.

1. Divide 2564 by 200.

2.00)25.64

12 1 Rem.
64

		Quot.	12	164 Rem.
2.	Divide 87654 by	600	*Ans.* 146	54 Rem.
3.	28347	80	354	27
4.	137000	1600	85	1000

Note. When the divisor is 10 the quotient will be had by cutting off one figure from the right of the dividend, when the divisor is 100 cut off two figures, when it is 1000 cut off three figures, &c. When the figures cut off from the right of the dividend are digits, they are to be considered as so much of a remainder.

EXAMPLE.

1. Divide 5640 by 10.

1.0)564.0 *Ans.* 564.

2.	Divide 25654	by 100	*Ans.* 256	54 Rem.
3.	876029	1000	876	29
4.	800000	10000	80	—

Application.

1. Several boys went to gather nuts, and collected 427 ; when they had divided them, each had 855; ho boys were in company?

2. If 3672 apple trees were planted in 28 rows, how many would there be in each row? *Ans.* 74.

3. If 45000 dollars were divided among 75 persons, how many would each one receive? *Ans.* 600.

4. Into how many parts must I divide the number 8164, so that each part may be 27, leaving the remainder 10?
Ans. 302.

5. There is a certain number, to the double of which if you add 12, then 5 times that sum will equal 150; what is that number. *Ans.* 9.

6. A father dying, left 13440 dollars to be divided among his 6 sons in the following manner, *viz.* to the eldest one-fourth part, to the second one-fifth, to the third one-sixth, to the fourth one-seventh, to the fifth one-eighth, and to the youngest the remainder; what was each son's share?
Ans. 1st 3360, 2d 2688, 3d 2240, 4th 1920, 5th 1680, 6th 1552 dolls.

7. What number, if multiplied by 72084, will make 5190048? *Ans.* 72.

8. A, B, and C, engage to do a piece of work for 228 dolls. which together they accomplish in 40 days; now it was previously agreed that A should have 10 cents per day more than B, and B 10 cents more than C; what was each man's share? *Ans.* A 80, B 76, C 72 dolls.

9. A man on counting his money, found he had an equal number of half eagles, (5 dollar pieces) half dollars, and quarter dollars, and that the whole amounted to 1437 dollars 50 cents; how many pieces of each kind had he?
Ans. 250 of each kind.

10. The crew of an armed ship, consisting of the captain, mate, and 40 men, took a prize worth 4550 dollars—now it was agreed that the captain should have 6 shares, the mate 4, and each seaman 1 share; what did each one receive? *Ans.* The capt. 546 dolls. the mate 364, and each seaman 91 dolls.

As but few examples are given under each of the foregoing rules, it is recommended that every teacher add as many similar ones, as may be found necessary to make the pupil well acquainted with their application, and both expert and accurate in working such questions as properly belong to these rules. Every experienced teacher is well aware that until this knowledge is obtained by the scholar, every attempt at any thing farther is only a waste of time and money. When this knowledge is once acquired, the future progress of the scholar will

be pleasant and rapid. The teacher will then be justly rewarded for his labor and trouble in this part, by the approbation of parents, and the gratitude of his scholars, who will have acquired the necessary qualifications (accuracy and expertness) for the great variety of studies and avocations in future life, which require the aid of arithmetic and mathematics.

PART II.

ARITHMETIC IN MIXED OR COMPOUND NUMBERS.

SECTION I.

FEDERAL MONEY

Is so called from its being the general currency established by the Federal, or United States' government, and is justly considered superior to every other kind of currency now in use for its simplicity and plainness.

Its denominations are,		Standard weight in troy dwt.	gr.	
10 Mills make	1 Cent			
10 Cents	1 Dime	1	16	Silver.
10 Dimes or 100 Cents	1 Dollar	17	1	Silver.
10 Dollars	1 Eagle	11	4	Gold.
	½ Eagle	5	14	Gold.

From this table it will readily be seen, that addition, subtraction, multiplication, and division of federal money may be performed as if they were whole numbers. It will also be seen that to reduce any number of mills to cents, it is only necessary to point, or cut off the last figure, as 100 mills =10,0 cents; and cents in the same way to dimes, as 100 cents=10,0 dimes, and dimes to dollars, as 100 dimes=10,0 dollars, and dollars to eagles, as 100 dollars=10,0 eagles; and also that eagles may be brought to dolls. and dolls. to dimes, &c. by adding a cipher to each one, as 10 E.=100 dolls.=1000d.=10000c.—100000m.

In all calculations in federal money, according to common *custom, it is usual* to omit the names of eagles, dimes,

and mills, and only to reckon by dollars and cents; t eagles being considered as so many 10 dollars, the dimes so many 10 cents, and the mills as fractional parts of t cent. See the following

TABLE.

Thousands of dollars	Hundreds of dollars	Eagles, or tens of dollars	Dollars	Dimes, or tens of cents	Cents	Mills, or parts of cents	Dolls.
			1,	2	5		1,25 cents
		3	4,	1	2	¼	34,12 and a fourth cents
	4	5	6,	2	5	½	456,25 and a half cents
8	2	6	4,	7	5	¾	8264,75 and three-fourth cents

Note. 1. In addition, subtraction, multiplication, and division federal money, if the sums are dollars only, the amount, remaind product, or quotient, will be dollars; but when the sums consist dollars and cents, or cents only, the two first right hand figures a cents, and all the rest are dollars.

2. When fractions of cents are used according to the above tab every four of them make one cent: in adding, or subtracting th we carry one for every four; and in multiplying, the upper figure, call the numerator, is to be multiplied by the multiplier, and divided by t lower figure, called the denominator.

EXAMPLES OF ADDITION.

E d d c m	*D c*	*D c*
25,6,4,8,2	5675,25	53258,75¼
24,7,6,2,4	2386,63	93620,33½
63,8,1,3,5	3972,80	30176,56¾
92,2,3,4,6	7285,75	27532,35
206,4,5,8,7		

SUBTRACTION.

E d d c m	*D c*	*D c*
83,6,5,3,5	653980,25½	365328,85¼
[illegible]	328763,31¼	238605,31½

MULTIPLICATION.

E d d c m	*D c*	*D c*
23,6,3,5,7	2637,25	6378,75½
3	6	9
70,9,0,7,1		

DIVISION.

E d d c m	*D c*	*D c*
2)63,3,8,6,2	5)3632,75	8)82750,33
31,6,9,3,1		

Promiscuous Questions.

1. Add 25 eagles, 62 dollars, 8 dimes, 75 cents, and 5 mills. *Ans.* 313d 55c 5m.

2. A person deposited at bank 1055 dollars in notes, 260 dollars in gold, 3650 dollars in silver, and 2,50 cents: how much is the amount? *Ans.* 4967d 50c.

3. Bought a barrel of sugar for 39 dollars 87½ cents, a bag of coffee for 22 dollars 18¾ cents, and a pound of tea for 2 dollars 12½ cents; how much do they all cost? *Ans.* 64d 18¾c.

4. Bought goods to the amount of 645 dollars 95¾ cents, and paid at the time of purchase 350 dollars; how much remains to be paid? *Ans.* 295d 95¾.

5. A man lent his friend 1000 dollars, and received at sundry payments, first 160 dollars 25 cents, second 285 dollars 66½ cents, third 300 dollars 28¾ cents; what remains yet to be paid? *Ans.* 253d 79¾c.

6. What is the product of 102 dollars 19 cents, multiplied by 120? *Ans.* 12262d 80c.

7. What will 16 barrels of flour amount to, at 4 dollars 50 cents per barrel? *Ans.* 72d.

8. How much will 132 pieces of calico come to, at 17 dollars 37½ cents a piece? *Ans.* 2293d 50c.

9. What is the quotient of 3022 dollars 50 cents, divided by 5? *Ans.* 1204d 50c.

10. A butcher bought 18 beef cattle for 252 dollars 90 cents; how much did he pay for each? *Ans.* 14d 05c.

1. Bought 45 yards of linen for 22 dollars 50 cents, what was the price of one yard? *Ans.* 50cts.

12. If 25 men expend 15555 dollars 50 cents in the erection of a bridge, how much has each one to pay, if the shares are equal? *Ans.* 622d 22c

Having treated of federal money separately, inasmuch as it requires to be well understood, seeing it is the general currency in the United States; we now proceed to the other parts of mixed numbers, or as they are frequently termed, divers denominations.

SECTION 2.

OF COMPOUND ADDITION.

COMPOUND Addition is the collecting together, and thereby ascertaining the amount of several quantities, of divers denominations.

RULE.

Place the numbers in such a manner, that all of the same denomination may stand directly under each other, then beginning with the lowest denomination, add as in whole numbers, carry at that number which will make one of the next greater; set down the remainder (if any) and so proceed till all are added.

PROOF as in simple addition.

ENGLISH MONEY.

The denominations are, pounds, shillings, pence and farthings, and are

Thus valued:

4 farthings (marked *qr.*) make	1 penny (marked)	*d.*
12 pence	1 shilling	*s.*
20 shillings	1 pound	£.

PENCE TABLE.

d.	*s.*	*d.*
20 pence make	1	8
30	2	6
40	3	4
50	4	2
60	5	0
70	5	10
80	6	8
. . . . .	7	6
. . . . .	8	4

TABLE OF SHILLINGS.

s.	£.	*s.*
20 shillings make	1	00
30	1	10
40	2	00
50	2	10
60	3	00
70	3	10
80	4	00
90	4	10
100	5	00

EXAMPLE.

£.	s.	d.	£.	s.	d.	qr.	£.	s.	d.	qr.
1256	11	8	35678	11	9	½	2368	17	5	½
9462	8	4	37562	18	7	¾	3969	19	11	¼
3215	10	6	63497	15	10	¼	9386	14	6	¾
£.13934	10	6								

TROY WEIGHT.

This weight is used for jewels, gold, silver, and liquors.

The denominations are, pounds, ounces, pennyweights, and grains.

Thus valued.

24 grains (*gr.*) make	1 pennyweight	*dwt.*
20 pennyweights	1 ounce	*oz.*
12 ounces	1 pound	*lb.*

EXAMPLES.

lb.	*oz.*	*dwt.*	*gr.*	*lb.*	*oz.*	*dwt.*	*gr.*	*lb.*	*oz.*	*dwt.*	*gr.*
4	10	15	16	5	8	11	16	4	5	17	11
8	6	10	11	9	10	15	21	9	6	12	9
6	9	14	23	6	11	18	17	18	11	19	23
20	3	1	2								

AVOIRDUPOIS WEIGHT.

This weight is used for heavy articles generally, and all metals but gold and silver.

The denominations are, tons, hundreds, quarters, pounds, ounces, and drams.

Thus valued.

16 drams (*dr.*) make	1 ounce	*oz.*
16 ounces	1 pound	*lb.*
28 pounds	1 quarter	*qr.*
4 quarters, (or 112 lb.)	1 hundred	*cwt.*
20 hundreds	1 ton	*T.*

EXAMPLES.

T.	*cwt.*	*qr.*	*lb.*	*oz.*	*dr.*	*T.*	*cwt.*	*qr.*	*lb.*	*oz.*	*dr.*
10	16	2	24	9	12	856	12	3	19	11	10
15	11	1	15	12	9	537	19	1	23	8	9
85	8	3	19	13	13	638	10	2	21	12	6
16	15	1	14	10	8	887	19	3	27	15	15

APOTHECARIES' WEIGHT.

This weight is used by apothecaries in mixing medicines, but they buy and sell by avoirdupois.

The denominations are, pounds, ounces, drams, scruples, and grains.

Thus valued.

20 grains (*gr.*) make . . .	1 scruple	*sc.* or	℈
3 scruples	1 dram	*dr.*	ʒ
8 drams	1 ounce	*oz.*	℥
12 ounces	1 pound	*lb.*	℔

EXAMPLES.

lb.	*oz.*	*dr.*	*sc.*	*gr.*	*lb.*	*oz.*	*dr.*	*sc.*	*gr.*
8	5	2	1	16	17	5	7	2	14
3	11	7	2	19	80	3	2	1	16
6	8	6	1	12	85	10	3	2	5
5	2	4	2	9	36	6	2	1	15

CLOTH MEASURE.

By this measure, cloths, ribands, &c. are measured.

The denominations are, Ells French, Ells English, Ells Flemish, yards, quarters, and nails.

Thus valued.

4 nails (*na.*) make . . .	1 quarter	*qr.*
4 quarters	1 yard	*yd.*
3 quarters	1 Ell Flemish	*E. Fl.*
5 quarters	1 Ell English	*E. En.*
6 quarters	1 Ell French	*E. Fr.*

EXAMPLES.

Yd.	*qr.*	*na.*	*E. Fl.*	*qr.*	*na.*	*E. Fr.*	*qr.*	*na.*	*E. En.*	*qr.*	*na*
56	2	2	80	2	3	16	4	2	53	4	3
86	1	3	18	1	2	17	5	1	53	3	2
33	3	2	36	2	1	80	2	2	32	2	1
38	2	1	36	2	1	13	3	3	81	0	0

LONG MEASURE.

By this, lengths and distances are measured.

The denominations are, degrees, leagues, miles, furlongs, poles, rods or perches, yards, feet, inches, and barley-corns.

Thus estimated.

3 barleycorns (*bc.*) make	1 inch	*in.*
12 inches	1 foot	*ft.*
3 feet	1 yard	*yd.*
5½ yards (or 16½ feet) .	1 rod, pole, or perch	*P.*
40 poles, rods, or perches, (or 220 yards)	1 furlong	*fur.*
8 furlongs (or 320 poles, or 1760 yds.)	1 mile	*M.*
3 miles	1 league	*L.*
60 geographic, or 69½ statute miles	1 degree	*deg.*

360 degrees make a circle, or the circumference of the earth.

A hand is a measure of 4 inches, and a fathom of 6 feet.

EXAMPLES.

deg.	*m.*	*fur.*	*po.*	*yds.*	*ft.*	*in.*	*bc.*
50	30	5	15	2	2	9	2
60	25	7	12	4	1	10	1
75	35	2	9	2	2	8	1
20	55	6	8	1	1	11	2

L.	*M.*	*fur.*	*yds.*	*ft.*	*in.*
5	2	6	75	2	11
3	1	4	95	1	9
2	1	3	15	2	8
1	2	5	200	1	6

LAND MEASURE.

By this the quantity of land is estimated.

The denominations are, acres, roods, perches, yards, and feet.

Thus rated.

9 feet (*ft.*) make	1 yard	*yd.*
30¼ yards	1 perch	*P.*
40 perches	1 rood	*R.*
4 roods (or 160 perches) . .	1 acre	

EXAMPLES.

A.	R.	P.	A.	R.	P.	A.	R.	P.
25	3	20	265	2	15	246	3	29
33	1	16	375	1	29	762	1	12
33	2	34	860	3	39	632	2	11
68	1	39	632	2	20	357	3	20

CUBIC, OR SOLID MEASURE.

By this, wood and other solid bodies are estimated.

The denominations are, cords, tons, yards, feet, and inches.

Thus rated:

1728 inches (*in.*) make . . .	1 foot	*ft.*
27 feet	1 yard	*yd.*
40 feet of round timber, or 50 feet of hewn timber	1 ton	*T.*
128 feet	1 cord	*cor.*

EXAMPLES.

Co.	ft.	in.	T.	ft.	in.	T.	ft.	in.
4	112	1260	6	39	1384	23	12	1400
6	84	1500	2	26	526	68	45	1600
8	127	1700	8	18	260	82	49	1700
5	63	403	3	12	1100	96	18	50

TIME.

This relates to duration.

The denomina[illegible] years, [illegible] days, hours, [illegible], and [illegible]

Th[illegible]

60 seconds ([illegible]
60 minutes
24 hours
7 days
4 weeks
12 [illegible]

[illegible]

N[illegible]
[illegible]

The number of days in the 12 *calendar months, is thus found:*

Thirty days are in September,
April, June, and November;
February hath twenty-eight alone,
And all the rest have thirty-one.

Note. Every fourth year is called bissextile, or leap-year, in which February has twenty-nine days.

EXAMPLES.

Y.	*M.*	*da.*	*h.*	*mi.*	*sec.*
22	10	25	16	34	55
34	6	16	20	48	33
46	9	13	23	59	59

Y.	*da.*	*h.*	*mi.*	*sec.*
4	350	15	19	5
2	268	13	54	38
6	350	22	50	50

MOTION.

This relates to the measure of circles.

The denominations are, circles, (or revolutions) signs, degrees, minutes, and seconds.

The relative differences are,

60 seconds (*sec.*) make	-	1 minute	*mi.* or ′
60 minutes - - - - -		1 degree	*deg.* or °
30 degrees - - - - -		1 sign	*sig.*
12 signs (or 360 degrees)		1 circle	

EXAMPLES.

sig.	*deg.*	*mi.*	*sec.*
2	24	48	58
2	29	59	59
3	21	20	20

sig.	°	′	″
3	20	30	40
2	25	35	45
3	26	38	58

LIQUID MEASURE.

This is used for measuring wine, spirits, cider, beer, &c.

The denominations are, tuns, pipes or butts, hogsheads, barrels, gallons, quarts, pints, and gills.

Thus estimated.

4 gills (*gi.*) make	- -	1 pint	*pt.*
2 pints - - - -		1 quart	*qt.*
4 quarts - - - -		1 gallon	*gal.*
63 gallons - - - -		1 hogshead	*hhd.*
2 hogsheads - - -		1 pipe, or butt	*pi. bt*
2 pipes (or four hogsheads)		1 tun	*T.*

Note. By a law of Pennsylvania, 32½ gallons make a barrel, and 16 gallons make a half barrel.

EXAMPLES.

T.	hhd.	gal.	qt.	pt.
4	3	53	2	1
6	2	25	3	1
8	1	62	1	1

T.	hhd.	gal.
24	2	33
19	3	54
34	1	50

DRY MEASURE.

This is used for measuring grain, salt, fruit, &c.

The denominations are, bushels, pecks, quarts, and pints.

Thus estimated.

2 pints (*pt.*) make	1 quart	*qt.*
8 quarts	1 peck	*P.*
4 pecks (or 32 quarts)	1 bushel	*bu.*

EXAMPLES.

bu.	P.	qt.
25	2	4
36	3	6
34	1	2
78	2	7

bu.	P.	qt.
256	3	6
243	1	6
468	3	1
584	2	7

bu.	P.	qt.
34156	3	7
2003	1	2
950	3	6
4809	0	0

SECTION 3.

OF COMPOUND SUBTRACTION.

COMPOUND SUBTRACTION is the taking a lesser number from a greater, of divers denominations, and thereby finding the difference.

RULE.

Set down the lesser number under the greater, as in compound addition. Then beginning with the lowest number, subtract as in subtraction of whole numbers: when the lower number is greater than the upper, take it from as many of that denomination as will make one of the greater, and to the remainder add the upper number, set down the amount and carry one to the next, and so proceed till all are ted.

PROOF.

Add the remainder to the lower line.

EXAMPLES.

	£.	s.	d.	qr.
From	256	15	6	½
Take	129	12	8	¾
Rem.	127	2	9	¾
Proof	256	15	6	½

	T.	cwt.	qr.	lb.	oz.	dr.
From	246	15	2	18	11	5
Take	89	16	1	24	8	15

	mi.	fur.	P.	ft.	in.	bc.
From	250	4	24	10	6	1
Take	125	6	30	5	10	2

	bu.	P.	qt.	pt
From	204	2	6	1
Take	150	3	2	0

	D.	h.	mi.	sec.
From	325	18	30	24
Take	236	20	45	50

	T.	hhd.	gal.	qt.	pt.
From	50	2	45	2	1
Take	20	3	60	3	0

	sig.	deg.	mi.	sec.
From	6	16	32	29
Take	3	24	16	48

	A.	R.	P.
From	1658	2	1[illegible]
Take	1249	3	[illegible]4

Promiscuous Questions in Compound Addition and Subtraction.

1. A merchant bought five pieces of linen, containing as follows: No. 1, 36 yards 3 quarters 2 nails; No. 2, 45 yards 1 quarter 3 nails; No. 3, 48 yards 2 quarters 1 nail; No. 4, 52 yards 3 nails; No. 5, 64 yards 2 quarters; how many yards were in all? *Ans.* 247*yds.* 2*qr.* 1*na.*

2. Sold 5 head of beef cattle, at the following prices, viz: the first for 6*l.* 2*s.* 4*d.* the second for 5*l.* 10*s.* 9½*d.* the third for 7*l.* the fourth for 8*l.* 10*s.* 6*d.* the 5th for 9*l.* 2*s.* 6*d.* and received 22*l.* 10*s.* 6*d.* in ready payment, and a note for the remainder; how much did the cattle cost, and for how much was the note given?

Ans. The cattle cost 36*l.* 6*s.* 1½*d.* and the note was for 13*l.* 15*s.* 7½*d.*

3. A silversmith bought 26*lb.* 9*oz.* 10*dwt.* of silver, and wrought up 18*lb.* 16*dwt.* 10*gr.* how much has he left?

Ans. 8*lb.* 8*oz.* 13*dwt.* 14*gr.*

4. A physician bought 6*lb.* 10*oz.* 6*dr.* 2*sc.* (apothecaries' weight) of medicine, and has used 4*lb.* 5*oz.* 4*dr.* 1*sc.* 17*gr.* what quantity has he yet remaining?

Ans. 2*lb.* 5*oz.* 2*dr.* 0*sc.* 3*gr.*

5. William was born on the 15th day of January, 1816, at 6 o'clock in the morning, and Charles was born on the 20th of March, 1817, at 9 in the evening; how much older is William than Charles? *Ans.* 1 year, 2mo. 5d. 15h.

6. An innkeeper bought four loads of hay, weighing as following, *viz.* first load, 18 hundred 2 quarters and 14 lb. second load, 16 hundred 3 quarters 18 lb. third load, 22 hundred and 24 lb. fourth load, 24 hundred and 1 quarter: how much hay in all? *Ans.* 4 tons 2 hundred.

7. From a piece of broadcloth which at first measured 55 yds. I sold to A 5½ yds. to B 6¼, to C 7¾, to D a quantity not recollected, and to E just half as much as to D; on measuring the remainder, I found there was 20½ yds. left; how many yards did D and E each receive?

Ans. D 10, and E 5 yds.

8. A wine merchant bought 1 pipe 2 hhds. and 3 qr. casks of wine, each 26 galls.; of these he sold 1 hhd. and 2 qr. casks; he also found that the pipe had leaked 17 galls. the remaining hogshead 11, and the cask 5½; how many gallons did he buy, and how many had he left?

Ans. Bought 330 galls. left 181½ galls.

9. Bought 4 pieces of cloth, the two first measured 9 Ells Fr. 3 qr. 2 na. each, and the two last 8 Ells Fr. 2 qr. 3 na. each, of these I sold 40¼ yards; how much have I left?

Ans. 13 y. 2 qr. 2 na.

SECTION 4.

OF COMPOUND MULTIPLICATION.

COMPOUND MULTIPLICATION teaches to multiply any given quantities or numbers of divers denominations.

Case 1.

When the multiplier does not exceed 12.

RULE.

Begin by multiplying the lowest number first, as in integers; divide the product by that number which will make one of the greater. Set down the remainder, if any, and carry the quotient to the next number.

EXAMPLES.

£.	s.	d.	qr.
24	10	6	½
			2
2)49	1	1	
24	10	6	½ Proof.

T.	cwt.	qr.	lb.	oz.	dr.
48	14	1	4	12	11
					3

lb.	oz.	dwt.	gr.
14	4	11	11
			5

bu.	pc.	qt.
24	3	7
		8

hhd.	gal.	qt.	pt.
25	48	3	1
			11

dg.	mi.	fur.	p.
8	24	6	34
			8

yds.	ft.	in.	bc.
24	2	8	2
			6

A.	R.	P.
89	3	26
		9

bu.	pc.	qt.
48	3	6
		11

d.	h.	mi.	sec.
24	19	38	15
			9

Y.	m.	w.	d.
125	8	3	4
			12

Application.

		£.	s.	d.		£.	s.	d.
1.	5 yards of cloth at	2	6	4	Ans.	11	11	8
2.	9 do. at	1	2	6¼		10	2	8¼
3.	11 bushels of flax-seed at		12	9½		7	0	8½
4.	12 do. clover-seed at	2	4	2½		26	10	6

Case 2.

When the multiplier exceeds 12, but is the exact product of any two *factors* in the table.

RULE.

Multiply the given sum by any one of these, in the same manner as above, and the product by the other.

EXAMPLES.

	£.	s.	d.	qr.	
Multiply	12	8	6	¼	by 18=3×6
				3	
	37	5	6	¾	
				6	
	223	13	4	½	

Application.

1. Multiply 4*T.* 3*cwt.* 1*qr.* 16*lb.* 8*oz.* 10*dr.* by 36.
Ans. 150*T.* 2*cwt.* 1*qr.* 7*lb.* 6*oz.* 8*dr.*
2. 120*l.* 6*s.* 9*d.* by 24. *Ans.* 2888*l.* 2*s.* 0*d.*
3. 24*T.* 4*cwt.* 2*qr.* 7*lb.* by 48.
Ans. 1162*T.* 19*cwt.* 0*qr.* 0*lb.*

Case 3.

When the multiplier is not the exact product of any two factors.

RULE.

Multiply as in the last case, by any two factors that will come the nearest to the multiplier, but less; and add for the deficiency.

EXAMPLES.

	bu.	*pc.*	*qt.*	
Multiply	12	2	4	by 17
				4×4+1=17
	50	2	0	
			4	
	202	0	0	=16
	12	2	4	= 1
	214	2	4	17

2. Multiply 8*D.* 4*h.* 12*mi.* 5*sec.* by 29.
Ans. 237*D.* 1*h.* 50*mi.* 25*sec.*

Case 4.

When the multiplier exceeds the product of any two factors in the table.

RULE.

Multiply by the units figure, as in case 1, and set down the amount; again multiply the given sum by the figure of tens, and that product by 10, and place this amount under the first; again multiply by the figure of hundreds, and the product by 10 and 10, which set down under the other products—in the same way for thousands, by three tens, &c.

EXAMPLES.

		s.	*d.*		2 6	2 6
Multiply		2	6	by 245	4	2
			5			
					10 0	5 0
		12	6	first, or units product	10	10
	5	0	0	second, or tens do.		
	25	0	0	third, or hundreds do.	5 0 0	2 10 0
						10
Ans.	30	12	6	total.		25 0 0

		£.	*s.*	*d.*			£.	*s.*	*d.*
2.	Multiply		14	6	by 240	*Ans.*	174	0	0
3.		1	2	3	117		130	3	3
4.		1	2	6	275		309	7	6

SECTION 5.

OF COMPOUND DIVISION.

Compound Division, teaches to divide any sum or quantity of divers denominations.

Case 1.

When the divisor does not exceed 12.

RULE.

Divide the highest, or left-hand denomination; if any remains, multiply by that number which will reduce it to the next highest, add this product to the second, then divide as before, and so proceed till all are divided.

PROOF—By compound multiplication.

EXAMPLES.

	£.	s.	d.	qr.
2)	465	10	6	½
Quotient	232	15	3	¼
				2
Proof	465	10	6	½

	£.	s.	d.	qr.
3)	563	15	4	½

	T.	cwt.	qr.	lb.
6)	91	16	1	14

	yds.	ft.	in.
5)	960	1	9

	T.	hhd.	gal.	qt.
8)	468	1	48	3

	w.	d.	h.	mi.	sec.
10)	30	6	18	48	50

Case 2.

When the divisor is the exact product of any two factors in the table.

RULE.

Divide first by one as above, and the quotient by the other.

EXAMPLES.

6) 224 12 6 by 30 = 6 × 5

5) 37 8 9

Quotient 7 9 9

	£.	s.	d.		£.	s.	d.
2. Divide	134	18	8	by 44	Ans. 3	1	4
3.	984	0	0	144	6	16	8
4.	474	0	0	72	6	11	8

Case 3.

When the divisor is not the exact product of any two factors in the table, or exceeds them.

RULE.

Divide the highest denomination in the given sum, in the same manner as in case 2, of whole numbers; reduce the *remainder*, if any, to the next lower denomination, adding it

to the number of the same denomination in the given sum; divide this in the same manner, and so proceed till all are divided.

EXAMPLES.

1. Divide 264*l.* 10*s.* 7½*d.* by 25.

25)264*l.* 10*s.* 7½*d.* (10*l.* 11*s.* 7½*d.* *Ans.*
25
14
20
25)290(11
25
40
25
15
12
25)187(7
175
12
4
25)50(2
50

	£.	s.	d.		£.	s.	d.
2. Divide	409	13	9	by 345	*Ans.* 1	3	9
3.	232	4	9	524		8	10¼
4.	3236	12	4½	654	4	18	11¾
5.	132	0	8	68	1	18	10

Promiscuous Questions, for exercise, in the foregoing rules of Compound Addition, Subtraction, Multiplication and Division.

1. What is the value of 672 yards of linen at 2*s.* 5*d.* per yard? *Ans.* 81*l.* 4*s.*

2. A goldsmith bought 11 ingots of silver, each of which weighed 4*lb.* 1*oz.* 15*dwt.* 22*gr.* how much do they all weigh? *Ans.* 45*lb.* 7*oz.* 15*dwt.* 2*gr.*

3. Bought 8 loads of hay, each weighing 1 ton 2 hundred 3 quarters 16 pounds; how much hay in all?

Ans. 9 ton 3 hun. 16lb.

4. Divide 9 ton 3 hundred 16lb. into 8 shares.

Ans. 1 ton 2 hun. 3qr. 16lb.

5. Bought 15 tracts of land, each containing 300 acres 2 roods and 20 perches; what is the amount of the whole?

Ans. 4509 acres 1 qr. 20 p.

6. Divide a tract of land containing 4509 acres 1 rood and 20 perches equally among 15 persons; what is each one's share? *Ans.* 300 acres 2 roods 20 perches.

7. Bought 179 bushels of wheat for 201 dollars 37½ cts. what is it per bushel? *Ans.* 1 doll. 12½ cents.

8. If a man spends 7 pence per day, how much will it amount to in a year? *Ans.* 10*l.* 12*s.* 11*d.*

9. What is the value of 1000 bushels of coal at 10½ cents per bushel? *Ans.* 105 dolls.

10. Bought 135 gallons of brandy at 1 dollar and 62½ cents per gallon, which was sold for 2 dollars and 5 cents per gallon; required the prime cost, what it was sold for, and the gain? *Ans.* prime cost 219 dolls. 37½ cts. sold for 276 dolls. 75 cts. gain 57 dolls. 37½ cts.

11. If 27 cwt. of sugar cost 47*l.* 12*s.* 10½*d.* what cost 1 cwt.? *Ans.* 1*l.* 15*s.* 3½*d.*

12. Suppose a man has an estate of 9708 dollars, which he divides among his four sons: to the eldest he gives ⅓, and to the other three an equal share of the remainder; what is the share of each?

Ans. eldest son, 3883 dolls. 20 cents, other sons, each 1941 dollars 60 cents.

13. A dollar weighs 17*dwt.* 8*gr.* what will 45 dollars weigh at that rate? *Ans.* 39*oz.*

14. An eagle of American gold coin should weigh 11*dwt.* 6*gr.*—now 150 were found to weigh 84*oz.* 7*dwt.* 20*gr.* how much was this over or under the just weight? *Ans.* 8*gr.* over.

15. What cost 2½ cwt. of sugar, at 13 cents 3 mills per pound? *Ans.* 37 dolls. 24 cts.

16. A merchant deposited in bank 35 twenty-dollar notes, 63 eagles, 284 dolls. 642 half dolls. 368 qr. dolls. 256 twelve and a half cent pieces; he afterwards gave a check to A for 560 dolls, and another to B for 820 dolls.; what sum has he still remaining in bank? *Ans.* 679 dolls.

17. A merchant bought a piece of broad cloth containing 38 yards, at 4 dolls. 66 cts. per yard; of this he found 4 yards were so damaged that he sold them at half price; 8 yards he sold at 5 dolls. 50 cents per yard: on the whole piece he gained 29 dolls. 56 cents: at what rate did he sell the remainder? *Ans.* 6 dollars per yard.

18. Five travellers, upon leaving a tavern in the morning, were charged 12½ cents each for their beds, 4 *um* for their supper and breakfast, 75 cents for

liquor among them all, 25 cents each for hay; the remainder of their bill, which amounted to 6 dollars, was for oats at $2\frac{1}{2}$ cents per quart; how many gallons of oats had they, and how much had each man to pay?

Ans. $8\frac{3}{4}$ gallons; each paid 120 cents.

19. A laborer engaged to work for 75 cents per day, working 8 hours each day, or 8 hours for a day's work; but being industrious he worked 12 hours 25 minutes each day for five days, and then 11 hours 30 minutes for 9 days more; what sum is he entitled to receive for his services?

Ans. 15 dollars 52 cents 3 mills+

20. If 25 hhds. contain 1534 galls. 1 qt. and 1 pt. of brandy, each an equal quantity, how much is there in each hogshead? *Ans.* 61 galls. 1 qt. 1 pt.

21. If a man do 114 hours 45 minutes' work in 9 days, how long did he work each day? *Ans.* 12*h.* 45*mi.*

22. Divide 180 dollars among 3 persons A, B, and C; give B twice as much as A, and C three times as much as B.

Ans. A 20 dolls. B 40 C 120

SECTION 6.

OF REDUCTION.

REDUCTION is the changing of numbers from one denomination to another, without altering their real value. Thus, 1 dollar, if reduced to cents, will be 100 cents, which in their real value are equal to 1 dollar: or, 3 feet reduced to yards, is one yard, which is still the same length as the 3 feet.

RULE.

If the reduction is from a *higher* to a *lower* denomination, multiply; but if from a *lower* to a *higher* denomination, divide by as many of the next less as make one of the greater; adding the parts of the same denomination to the product as it descends; and setting down the remainders as it ascends.

Reduction ascending, and descending, mutually prove each other.

A TABLE of the Weight and Value of Coins as they pass in the respective States of the Union, with their Sterling, and Federal Value.

Names of Coins.	*Standard Weight.*		*Sterling Money of Great Britain.*			*N. Hampshire, Massachusetts, Rhode Island, Connecticut, & Virginia.*			*New York and North Carolina.*			*New Jersey, Pennsylvania, Delaware, and Maryland.*			*South Carolina and Georgia.*			*Federal value.* Eagle. Dolls. Dimes Cents. Mills.				
(GOLD.)	*dwt.*	*gr.*	£.	*s.*	*d.*	£.	*s.*	*d.*	£.	*s.*	*d.*	£.	*s.*	*d.*	£.	*s.*	*d.*	E.	*d.*	*d.*	*c.*	*m.*
A Johannes, . . .	18	0	3	12	0	4	16	0	6	8	0	6	0	0	4	0	0	1	6,	0	0	0
A half Johannes, . .	9	0	1	16	0	2	8	0	3	4	0	3	0	0	2	0	0		8,	0	0	0
A Doubloon, . . .	16	21	3	6	0	4	8	0	5	16	0	5	12	6	3	10	0	1	4,	9	3	3
A Moidore, ...	6	18	1	7	0	1	16	0	2	8	0	2	5	0	1	8	0		6,	0	0	0
An English Guinea, .	5	6	1	1	0	1	8	0	1	17	0	1	15	0	1	1	9		4,	6	6	7
A French Guinea, .	5	5	1	1	0	1	7	6	1	16	0	1	14	6	1	1	5		4,	6	0	0
A Spanish Pistole, .	4	6	0	16	6	1	2	0	1	9	0	1	8	0	0	18	0		3,	7	7	3
A French Pistole, . .	4	4	0	16	0	1	2	0	1	8	0	1	7	6	0	17	6		3,	6	6	7
(SILVER.)																						
An English or French Crown,	19	0	0	5	0	0	6	8	0	8	9	0	8	3	0	5	0		1,	1	0	0
The Dollar of Spain, Sweden, or Denmark,	17	8	0	4	6	0	6	0	0	8	0	0	7	6	0	4	8		1,	0	0	0
An English Shilling, .	3	18	0	1	0	0	1	4	0	1	9	0	1	8	0	1	0		0,	2	2	2
A Pistareen, ...	3	11	0	0	10¾	0	1	2	0	1	7	0	1	6	0	0	11		0,	2	0	0

All other gold coins, of equal fineness, at 89 cents *per dwt.* and silver at 111 cents *per oz.*

A TABLE of other Foreign Coins, &c. with their value in Federal Money, as established by a late act of Congress.

	E.	d.	d.	c.	m.
Pound Sterling	0	4,	4	4	4
Pound of Ireland	0	4,	1	0	0
Pagoda of India	0	1,	9	4	0
Tale of China	0	1,	2	4	0
Millree of Portugal	0	1,	2	4	0
Ruble of Russia	0	0,	6	6	0
Rupee of Bengal	0	0,	5	5	5

	E.	d.	d.	c.	m.
The Guilder of the U. Netherlands	0	0,	3	9	0
Mark Banco of Hamburg	0	0,	3	3	5
Livre Turnois of France	0	0,	1	8	5
Real Plate of Spain	0	0,	1	0	0

MONEY.

Cents are reduced to *pence* by subtracting one-tenth of their number. Pence are reduced to cents by adding one-ninth of their number.

Pence are to cents as 9 is to 10, and to mills as 9 to 100. This only applies where the dollar passes at 7*s*. 6*d*. or 90 pence.

1. Reduce 100 cents to pence.

10) 100 cents
10

90 pence. *Ans.*

2. Reduce 90 pence to cents.

9) 90 pence
10

100 cents. *Ans.*

3. Reduce 125*l*. 10*s*. 6½*d*. to farthings.

125*l*. 10*s*. 6½*d*.
20

2510 shillings
12

30126 pence
4

120506 farthings. *Ans.*

4. Reduce 120506 farthings to pounds.

Ans. 125*l*. 10*s*. 6½*d*.

5. Reduce 260 cents to pence. *Ans.* 234*d*.

6. Reduce 480*l.* 19*s.* 9*d.* to cents.

Ans. 128263½ cents.

7. Reduce 4658 pence to pounds. *Ans.* 19*l.* 8*s.* 2*d.*

8. Reduce 648 pence to cents. *Ans.* 720 cents.

9. Reduce 720 cents to pence. *Ans.* 648 pence.

10. Reduce 24235 half-pence to pounds.

Ans. 50*l.* 9*s.* 9½*d.*

11. How many pounds, Pennsylvania currency, in 216 French crowns? *Ans.* 89*l.* 2*s.*

12. In 29*l.* 17*s.* how many cents and dollars?

Ans. 7960 cents—79 dolls. 60 cts.

13. In 375*l.* Pennsylvania currency, how many dollars?

Ans. 1000 dolls.

Note. To bring pounds (Penn. currency) to dollars, multiply by 8 and divide by 3; and dollars to pounds, multiply by 3 and divide by 8.

TROY WEIGHT.

1. Reduce 115200 grains to pounds. *Ans.* 20*l.*

2. Reduce 30*lb.* to grains. *Ans.* 172800*gr.*

3. Reduce 45648 pennyweights to ounces.

Ans. 2282*oz.* 8*dwt.*

4. Reduce 4*lb.* 8*oz.* 15*dwt.* 20*gr.* to grains.

Ans. 27260*gr*

5. Reduce 27260 grains to pounds.

Ans. 4*lb.* 8*oz.* 15*dwt.* 20*gr.*

6. In 24 spoons, each weighing 8*dwt.* 6*gr.* how many grains? *Ans.* 4752*gr.*

AVOIRDUPOIS WEIGHT.

1. Reduce 3 tons to pounds. *Ans.* 6720*lb.*

2. Reduce 2867200 drams to tons. *Ans.* 5 tons.

3. Reduce 5 tons to drams. *Ans.* 2867200*dr.*

4. In 6 barrels of flour, each weighing 1*cwt.* 3*qr.* how many pounds? *Ans.* 1176*lb.*

5. In 16*cwt.* 2*qr.* 14*lb.* how many pounds?

Ans. 1862*lb.*

6. In a load of hay weighing 2876*lb.* how many hundreds? *Ans.* 25*cwt.* 2*qr.* 20*lb.*

APOTHECARIES' WEIGHT.

1. Reduce 15*lb.* to scruples. *Ans.* 4320*sc.*

2. In a bottle containing 3*lb.* of calomel, how many grains? *Ans.* 17280*gr.*

3. In 2½*lb.* of drugs, how many parcels, each 16 drams? *Ans.* 15 parcels.

4. In 576000 grains, how many pounds? *Ans.* 100*lb.*

CLOTH MEASURE.

1. Reduce 250 yards to nails. *Ans.* 4000 nails.

2. In 8642 nails, how many Ells English? *Ans.* 432 Ells E. 2 nails.

3. In 324 Ells French, how many yards? *Ans.* 486 yards.

4. In 16 bales of cloth, each measuring 36 Ells Flemish, how many yards? *Ans.* 432 yards.

LONG MEASURE.

1. Reduce 260 miles to inches. *Ans.* 16473600 inches.

2. Reduce 11 miles 7 furlongs 38 perches 2 yards 2 feet, to barley-corns. *Ans.* 2280060*bc.*

3. Reduce 1267200 feet to geographical degrees. *Ans.* 4 degrees.

4. Reduce 3 leagues 2 furlongs 110 yards 1 foot 5 inches, to inches. *Ans.* 590057 inches.

5. How many inches will reach round the world, at 60 miles to a degree? *Ans.* 1368576000 inches.

LAND MEASURE.

1. Reduce 25 acres to perches. *Ans.* 4000 perches.

2. Reduce 176000 perches to acres. *Ans.* 1100 acres.

3. A tract of land containing 640000 perches is to be divided into 400 equal shares; how many acres will be in each share? *Ans.* 10 acres.

4. In 10 acres, how many square inches? *Ans.* 62726400 inches.

CUBIC, OR SOLID MEASURE.

1. Reduce 3200 feet of wood to cords.

Ans. 25.

2. In 20 tons of square timber, how many feet?

Ans. 1000 feet.

3. In 30 tons of round timber, how many inches?

Ans. 2073600 inches.

Note. The cubic foet of any circular body, such as grindstones, &c is found in the following manner. Add half the diameter to the whole diameter; multiply the amount by the aforesaid half, and this product by the thickness; this will give the contents in cubic inches; divide these by 1728, and the quotient will be the cubic feet.

4. In a grindstone 48 inches diameter and 6 inches thick, how many feet?

48 diameter
24 half do.

72
24

288
144

1728
6

1728) 10368 (6 cubic feet. *Ans.*
10368
---

5. In a millstone 4 feet 6 inches diameter, and averaging 18 inches in thickness, how many cubic feet?

54
27

81
27

567
162

2187
18

17496
2187

1728) 39366 (22 feet 1350 inches. *Ans.*
3456

4806
3456

1350

TIME.

1. Reduce 8 weeks 2 days 6 hours 20 minutes, to minutes.
Ans. 83900 minutes.
2. Reduce ten years to seconds.
Ans. 315576000 sec.
3. How many days since the commencement of the Christian era to the present time, 1823?
Ans. 665850 days 18 hours.
4. How many seconds in a week? *Ans.* 604800 sec.

LIQUID MEASURE.

1. Reduce 4 tuns to pints. *Ans.* 8064 pints.
2. Reduce 4032 pints to hogsheads. *Ans.* 8 hhds.
3. Reduce 38 hogsheads to pints. *Ans.* 19152 pints.

DRY MEASURE.

1. Reduce 78 bushels 3 pecks 7 quarts to pints.
Ans. 5054 pints.
2. Reduce 2196 pints to bushels. *Ans.* 34*bu.* 1*pc.* 2*qt.*

PART III.

DECIMAL ARITHMETIC.

DECIMAL ARITHMETIC is a plain and easy method of discovering the value of an unit, or one, divided into any given number of parts. Thus, if 1 dollar is divided into 10 equal parts, any one of these parts will be one-tenth, 2 will be two-tenths, 3, three-tenths, &c. Again, if 1 dollar is divided into a hundred equal parts, any one of these will be one-hundredth, 2, two-hundredths, &c.

The number of parts into which the unit is divided is called the *denominator*, and any number of these parts less than the whole is called the *numerator*, and which always stands over the denominator; thus,

$$\frac{\text{2 numerator}}{\text{10 denominator}}$$

is read two-tenths; and these two so placed constitute what is termed a *fraction*. In decimal fractions, the denominator can only be an unit, with one or more ciphers

added thereto; as $\frac{5}{10}$, $\frac{25}{100}$, $\frac{225}{1000}$. The numerators of these are usually written without their denominators, and are distinguished from whole numbers, by prefixing a point called the separatrix, as ,5 ,25 ,225.

Ciphers placed to the right hand of decimals, make no change in their value, for ,5 ,50 ,500, &c. are decimals of the same value, each being equal to $\frac{1}{2}$. But when prefixed to the decimal, they decrease the value in a tenfold proportion. Thus, ,5 ,05, ,005, have the same proportion to each other as 5, 50, 500, have in whole numbers.

This is made plain by the following

TABLE.

Integers.							*Decimals.*						
						5							
					5	0	,5						
				5	0	0	,0	5					
			5	0	0	0	,0	0	5				
		5	0	0	0	0	,0	0	0	5			
	5	0	0	0	0	0	,0	0	0	0	5		
5	0	0	0	0	0	0	,0	0	0	0	0	5	
Millions	Hundreds of thou.	Tens of thousands	Thousands	Hundreds	Tens	Units	Tenths	Parts of a hundred	Parts of a thousand	Parts of ten thousand	Parts of a hund. thou.	Parts of a million	

SECTION I.

ADDITION OF DECIMALS.

RULE.

Set down the numbers according to their value, *viz.*—units under units, tenths under tenths, &c. Then **add as** in addition of whole numbers, and place the point in the sum exactly under those in the given sum.

EXAMPLES.

2468,5036	3460000,0000643
521,0428	460000,000643
32,0004	3400,3680005.
3021,5468	

3. Add 283,604 + 490006,003275 + 21,05 + 1,2 + 6200,3476. *Ans.* 496512,204875.

4. Add ,246+,012+,02+,6+,413+,5. *Ans.* 1,791.

5. Add 25,52+225,005+,0035+844+2,2+300,825 +,00005. *Ans.* 1397,55355.

6. Add one hundred and twenty-five, and five-tenths,+ ten thousand, and five millionths,+ fifteen, and seventy-two thousandths,+ two, and one hundredth.

Ans. 10142,582005.

7. Add five, and four-tenths,+ fifteen and four hundredths, +one hundred, and four thousandths,+ six thousand and four hundred thousandths,+ ninety-three thousand eight hundred and eighty, and four ten thousandths.

Ans. 100000,44444.

SECTION 2.

SUBTRACTION OF DECIMALS.

RULE.

SET the less under the greater, with the points as in addition, and place the point in the remainder, in the same manner.

EXAMPLES.

From 6432,50437	From 84[illegible],045	From 15,6547
Take 369,95429	Take 162,540368	Take 7,35
Rem. 6062,55008		

4. From 45,005 take 23,65482. *Ans.* 21,35018.

5. From six hundred and twenty, and two-tenths, take two hundred and two thousandths. *Ans.* 420,198

6. From 5 take ,10438. *Ans.* 4,89562.
7. From 2 take ,00002. *Ans.* 1,99998.
8. From sixteen take sixteen thousandths parts. *Ans.* 15,984.

SECTION 3.

MULTIPLICATION OF DECIMALS.

RULE.

MULTIPLY as in whole numbers, and from the product point off as many on the right-hand for decimals, as there are in both the factors. If the whole product should be too few, then must ciphers be added on the left of the product till an equal number is had.

EXAMPLES.

Multiply	29,831	24,021	22,2043
by	,952	4,23	,12345
	59662	72063	1110215
	149155	48042	888172
	268479	96084	666129
			444086
Product	28,399112	101,60883	222043
			2,741120835

4. Multiply ,385746 by ,00463 *Ans.* ,00178600398
5. 158,694 23,15 3673,7661
6. ,024653 ,00022 ,00000542366

7. Multiply twenty-five and four hundredths, by two thousandths. *Ans.* ,05008.

8. Multiply six hundred and forty-five, and three thousandths, by five millionths. *Ans.* ,003225015.

Note. The product of any number when multiplied by a decimal only, will be less than the multiplicand, in the same proportion as the *multiplier* is less than one.

CONTRACTION IN MULTIPLICATION OF DECIMALS.

If only a limited number of decimals is sought for, instead of retaining the whole product, obtained by the foregoing method, work by the following

RULE.

1. Set the multiplier, *in an inverted order*, under the multiplicand, placing the *units* figure of the multiplier under the lowest decimal place in the multiplicand, that is wished to be retained.

2. In multiplying, omit those figures in the multiplicand which are on the right of the multiplying figure, but to the first figure in each line of the product, add the carriage which would arise from the multiplication of the omitted figures, carrying one from 5 to 15, 2 from 15 to 25, 3 from 25 to 35, &c. Place the first figures in each product directly under each other, and add as in addition.

Note. If you would be absolutely certain that the last figure retained is the *nearest* to the truth, work for *one* place more than you wish to retain.

EXAMPLES.

1. Multiply 34,6733 by 3,1416, retaining four decimal places in the product.

34,6733
61413 inverted

1040199
34673
13869
347
208

108,9296 *Ans.*

2. Multiply ,78543 by ,346787, retaining five decimal places in the product.

346787		Or thus,	,78543
34587	inverted		787643,0
24275			23563
2774			3142
173			471
14			55
1			6
,27237	*Ans*		,27237

3. Multiply 23,463 by 2,34, retaining three decimals.
Ans. 54,903.

4. Multiply 234,216 by 2,345, retaining two decimals.
Ans. 549,23.

5. Multiply 3,141592 by 52,7438, retaining four decimals. *Ans.* 165,6995.

SECTION 4

DIVISION OF DECIMALS.

RULE.

DIVIDE in the same manner as in whole numbers, and point off on the right of the quotient as many figures for decimals, as the decimals in the dividend exceed those in the divisor. When the decimals in the divisor exceed those in the dividend, let ciphers be added to the dividend, till they equal those in the divisor. And if there be a remainder, let ciphers be annexed thereto, and the quotient carried on to any degree of exactness.

EXAMPLES.

29,831) 28,399112 (,952	24,021) 101,60883 (4,23
268479	96084
155121	55248
149155	48042
59662	72063
59662	72063

Note. When the divisor is 10, 100, 1000, &c. the division is performed by pointing off as many figures in the dividend for decimals, as there are ciphers in the divisor.

Thus, 6856 divided by $\left\{\begin{matrix}10\\100\\1000\end{matrix}\right\}$ is $\left\{\begin{matrix}685,6\\68,56\\6,856\end{matrix}\right\}$

3.	Divide 65321	by 23,7	*Ans.* 2756,16+
4.	234,70525	64,25	3,653
5.	10	3	3,3333+
6.	9	,9	10
7.	,00178600398	,00463	,385746

8.	Divide	,2327898	by	2,46	*Ans.* ,09463
9.		,2327898		,09463	2,46
10.		,000162		,018	,009

CONTRACTION IN DIVISION OF DECIMALS.

When only a limited number of decimals in the quotient is sought for, work by the following

RULE.

1. Take as many figures only on the left hand side of the divisor, as the whole number of figures sought for in the quotient, and cut off the rest.

2. Make each remainder a *new dividend*, and for a *new divisor*, point off *one* figure continually from the right hand of the former divisor, taking care to bring in the increase, or carriage of the figures so cut off, as in multiplication.

Note. When the whole divisor does not contain as many figures as are sought for in the quotient, proceed as in common division, without cutting off a figure, till the figures in the divisor shall equal the remaining figures required in the quotient, and then begin to cut off as above directed.

EXAMPLES.

1. Divide 14169,206623851 by 384,672258, retaining four decimal places in the quotient, or in all six quotient figures.

3.8.4,6.7.2|258) 14169,206623851 (36,8345. *Ans.*
1154017

262903
230803

32100
30774

1326
1154

172
153

19
19

2. Divide ,07567 by 2,32467, true to four decimal pla or three significant figures, the first being a cipher.

2,3.2|467),07567 (,0326. *Ans.*
697
59
46
13
14

3. Divide 5,37341 by 3,74, true to four decimal place

3,74) 5,37341 (1,4367. *Ans.*
374
1633
1496
1374
1122
252
224
28
26

4. Divide 74,33373 by 1,346787, true to three deci places. *Ans.* 55,19
5. Divide 87,076326 by 9,365407, true to three deci places. *Ans.* 9,29
6. Divide 32,68744231 by 2,45, true to two deci places. *Ans.* 13,3
7. Divide ,0046872345 by 6,24, true to five decir places. *Ans.* ,0007

SECTION 5.

REDUCTION OF DECIMALS.

Case 1.

To reduce a *vulgar* fraction to a decimal.

RULE.

Annex one or more ciphers to the numerator, and divide by the denominator; the quotient will be the answer in decimals.

EXAMPLE.

1. Reduce $\frac{1}{4}$ to a decimal.

4)1,00

,25 *Ans.*

2. Reduce $\frac{1}{2}$ to a decimal. *Ans.* ,5
3. - $\frac{3}{4}$ to a decimal. ,75
4. - $\frac{7}{8}$ to a decimal. ,875
5. - $\frac{1}{25}$ to a decimal. ,04
6. - $\frac{57}{60}$ to a decimal. ,95
7. - $\frac{6}{15}$ of a dollar to cents. ,40 cts.

Case 2.

To reduce numbers of different denominations to a decimal of equal value.

RULE.

Set down the given numbers in a perpendicular column, having the least denomination first, and divide each of them by such a number as will reduce it to the next name, annexing the quotient to the succeeding number; the last quotient will be the required decimal.

EXAMPLE.

1. Reduce 17*s.* 8¾*d.* to the decimal of a pound.

4	3
12	8,75
20	17,720166

,8864583+ *Ans.*

2. Reduce 19*s.* to the decimal of a pound. *Ans.* ,95
3. - 3*d.* to the decimal of a shilling. ,25
4. - 3*d.* to the decimal of a pound. ,0125
5. - 4*cwt.* 2*qr.* to the decimal of a ton. ,225
6. - 2*qr.* 14*lb.* to the decimal of a cwt. ,625
7. - 3*qr.* 3*na.* to the decimal of a yard. ,9375

Case 3.

To reduce a decimal to its equal value in integers.

RULE.

Multiply the decimal by the known parts of the integer.

EXAMPLE.

1. Reduce ,8864583 of a pound to its equivalent value in integers.

,8864583
20

s. 17,7291660
12

d. 8,7499920
4

qr. 2,9999680

It is usual when the left hand figure in the remaining decimal exceeds five, to expunge the remainder, and add one to the lowest integer. Thus, instead of 17*s.* 8*d.* 2,999, &c. we may say 17*s.* 8¾*d.* *Ans.*

2. What is the value of ,75 of a pound? *Ans.* 15*s.*
3. What is the value of ,7 of a pound troy?
Ans. 8*oz.* 8*dwt.*
4. What is the value of ,617 of a cwt.?
Ans. 2*qr.* 13*lb.* 1 *oz.* 10+*dr.*
5. What is the value of ,3375 of an acre?
Ans. 1 *rood*, 14*per.*
6. What is the value of ,258 of a tun of wine?
Ans. 1*hhd.* 2+*gals.*
7. What is the proper quantity of ,761 of a day?
Ans. 18*h.* 15*mi.* 50,4*sec.*
8. What is the proper quantity of ,7 of a *lb.* of silver?
Ans. 8*oz.* 8*dwt.*
9. What is the proper quantity of ,3 of a year?
Ans. 109*d.* 13*h.* 48*mi.*
10. What is the difference between ,41 of a day and ,16 of an hour? *Ans.* 9*h.* 40*mi.* 48*sec.*
11. What is the sum of ,17*T.* 19*cwt.* ,17*qr.* and 7*lb.*?
Ans. 3*cwt.* 2*qr.* 15,54*lb.*

Promiscuous Questions in Decimal Fractions.

1. Multiply ,09 by ,009. *Ans.* ,00081.
2. In ,36 of a ton (avoirdupois) how many ounces?
Ans. 12902,4*oz.*
3. What is the value of ,9125 of an ounce troy?
Ans. 18*dwt.* 6*gr.*
4. Reduce $\frac{4}{31\frac{1}{2}}$ to a decimal. *Ans.* ,0127 nearly.
5. Reduce 2*oz.* 16*dwt.* 20*gr.* to the decimal of a pound troy. *Ans.* ,2368+nearly.
6. What is the length of ,1392 of a mile?
Ans. 1 fur. 4 per. 3 yds nearly.
7. What multiplier will produce the same result, as multiplying by 3, and dividing the product by 4?
Ans. ,75.
8. What decimal of 1*cwt.* is 6*lb.* *Ans.* ,053571+.
9. What part of a year is 109 days 12 hours?
Ans. ,3.
10. In ,04 of a ton of hewn timber, how many cubic inches? *Ans.* 3456.
11. What is the value of $\frac{3}{13}$ of a dollar divided by 3?
Ans. 6$\frac{2}{3}$ cents.
12. What is the value of ,875 of a hhd. of wine?
Ans. 55 gal. 0 qt. 1 pt.
13. What divisor, true to six decimal places, will produce the same result as multiplying by 222?
Ans. ,004504.
14. In ,05 of a year, how many seconds, at 365 days 6 hours to the year? *Ans.* 1577880.
15. What number as a multiplier will produce the same result as multiplying by ,73 and dividing first by 3, and the quotient by ,25? *Ans.* ,97$\frac{1}{3}$.
16. What is the difference between ,05 of a year, and ,5 of an hour? *Ans.* 2*w.* 2*d.* 18*h.* 42*m.*
17. In ,4 of a ton, ,3 of a hhd. and ,8 of a gallon, how many pints? *Ans.* 964.
18. How many perches in ,6 of an acre; multiplied by ,02? *Ans.* 1,92.
19. What part of a cord of timber is 1 cubic inch?
Ans. ,000004+
20. What part of a circle is 28 deg. 48 minutes?
Ans. ,08.

PART IV.

PROPORTIONS.

THIS part of arithmetic which treats of proportions is very extensive and important. By it an almost innumerable variety of questions are solved. It is usually divided into three parts, *viz.* Direct, Inverse, and Compound.—The first of these is called the *Single Rule of Three Direct*, and sometimes by way of eminence the *Golden Rule*. The second is called the *Single Rule of Three Inverse:* and the last is called the *Double Rule of Three.* In all these, certain numbers are always given, called *data*, by the multiplication and division of which, the answer in an exact ratio of proportion to the other terms is discovered.

SECTION 1.

SINGLE RULE OF THREE DIRECT

IN this rule three numbers are given to find a fourth, that shall have the same proportion to the third, as the second has to the first.

If by the terms of the question, *more* requires *more*, or *less* requires *less*, it is then said to be direct, and belongs to this rule.

In stating questions in this rule, the middle term must always be of the same name with the answer required; the last term is that which asks the question, and that which is of the same name as the demand, the first. When the question is thus stated, reduce the first and third terms to the lowest denomination in either; and the middle term (if compound) to its lowest, and proceed according to the following

RULE.

Multiply the second and third terms together, and divide the product by the first; the quotient will be the fourth term, or answer, in the same name with the second.

PROOF.

Invert the question, making the answer the first term; the result will be, the first term in the original question.

Note. 1. After division if there be any remainder, and the quotient be not in the lowest denomination, it must be reduced to the next less denomination, dividing as before, till it is brought to the lowest denomination, or till nothing remains.

2. When any of the terms are in *federal money*, the operation is conducted in all respects as in simple numbers, taking care to place the separatrix between dollars and cents, according to what has already been laid down in federal money and decimal fractions.

EXAMPLE.

1. If 8 yards of cloth cost 32 dollars, what will 24 yards cost?

Yds.	*D.*		*Yds.*			*D.*	*Yds.*		*D.*
As 8	: 32	::	24		Proof.	As 96	: 24	::	32
	24						32		
	128						48		
	64						72		
	8)768						96)768(8		
	96 *Ans.*						768		

2. When sugar is sold at 12 dollars 32 cts. per cwt. what will 16*lb.* cost? *Ans.* 1 doll. 76 cts.

3. What is the amount of 3 *cwt.* of coffee at 36 cents per pound? *Ans.* 120 dolls. 96 cts.

4. What will 4 pieces of linen come to, containing 23, 24, 25, and 27 yards, at 72 cents per yard?
Ans. 71 dolls. 28 cts.

5. What will 4*cwt.* 2*qr.* 8*lb.* of iron come to at 48 cents for 4*lb.*? *Ans.* 61 dolls. 44 cts.

6. What will 128*lb.* of pork come to at 8 cts. per pound?
Ans. 10 dolls. 24 cts.

7. If 9½ dozen pair of stockings, cost 68 dollars 40 cents what will 3 pair cost? *Ans.* 1 doll. 80 cts.

8. If 20 bushels of oats cost 9 dollars 60 cents, what will three bushels come to? *Ans.* 1 doll. 44 cts.

9. A merchant bought a piece of cloth for 16 dollars 50 cents, at 75 cents per yard; how many yards were there in the piece? *Ans.* 22 yds.

10. If 17*cwt.* 3*qr.* 17*lb.* of sugar cost 320 dollars 80 cts. what must be paid for 6*oz.*? *Ans.* $

11. If 9,7*lb.* of silver is worth 97 dollars, what is the value of 1,5*oz.*? *Ans.* 1 doll. 25 cts.

12. If 125,5 acres are sold for 627,5 dollars, what will 4,75 acres cost? *Ans.* 23 dolls. 75 cts.

13. If 1,5 gallons of wine cost 4 dollars 50 cents, what will 1,5 tuns cost? *Ans.* 1134 dolls.

14. How many reams of paper at 1 dollar 66 cents, 1 dollar 97 cents, and 2 dollars 31 cents per ream may be purchased for 528 dollars 66 cents, of each an equal number? *Ans.* 89 reams of each sort.

15. When iron is sold for 224 dollars per ton, what will 1*qr.* 14*lb.* cost? *Ans.* 4 dolls. 20 cts.

16. A merchant paid 1402 dollars 50 cents for flour, at 5 dollars 50 cents per barrel; how many barrels must he receive? *Ans.* 255 barrels.

17. A man has a yearly salary of 1186 dollars 25 cents, how much is it per day? *Ans.* 3 dolls. 25 cts.

18. A man spends 2 dollars 25 cents per day, and saves 378 dollars 75 cents at the end of the year, what is his yearly salary? *Ans.* 1200 dolls.

19. What will 4*T.* 10*cwt.* 1*qr.* 12*lb.* of hay come to at 1 dollar 12 cents per cwt.? *Ans.* 101 dolls. 20 cts.

20. How much will a grindstone 4 feet 6 inches diameter, and 9 inches thick, come to at 1 dollar 10 cents per cubic foot? *Ans.* 12 dolls. 53 cts.

21. What will a grindstone 28 inches diameter, and 3,5 inches thick, come to at 1 dollar 90 cents per cubic foot? *Ans.* 2 dolls. 26 cts.

22. At 22*l.* 8*s.* per ton, what will 203*T.* 9*cwt.* 3*qr.* 3*lb.* of tobacco come to? *Ans.* 4558*l.* 3*s.*

23. If 850 dolls. 50 cents is paid for 18 pieces of cloth at the rate of 11 dollars 25 cents for 5 yards, how many yards were in each piece, allowing an equal number to each piece? *Ans.* 21 yds.

24. If 12½ yards of muslin cost 1*l.* 17*s.* 6*d.* what is it per yard? *Ans.* 3*s.*

25. If a staff 4 feet long cast a shadow (on level ground) 7 feet long, what is the height of a steeple whose shade at the same time, is 218 feet 9 inches? *Ans.* 125 feet.

26. If 4292 dollars 32½ cents are paid for 476 acres 3 *rds 28 perches* of land, how much is it per acre?

Ans. 9 dollars.

annual income be 1333 dollars, and he

expend daily 2 dollars 14 cents, how much will he save at the end of the year? *Ans.* 551 dolls. 90 cts.

28. If 321 bushels of wheat cost 240 dollars 75 cents, what is it per bushel? *Ans.* 75 cts.

29. If $1\frac{1}{2}$ yard of cloth cost 2 dollars 50 cents, what will 1 quarter 2 nails come to? *Ans.* $62\frac{1}{2}$ cts.

30. Bought 3 pipes of wine, containing $120\frac{1}{2}$, 124, and $126\frac{3}{4}$ gallons, at 5*s.* 6*d.* per gallon; what do they cost?

Ans. 102*l.* 1*s.* $10\frac{1}{2}$*d.*

31. A sets out from a certain place and goes 12 miles a day; 5 days after, B sets out from the same place, the same way, and goes 16 miles a day; in how many days will he overtake A? *Ans.* 15 days.

32. If I have owing to me 1000*l.* and compound with my debtor, at 12*s.* 6*d.* per pound, how much must I receive?

Ans. 625*l.*

33. If 365 men consume 75 barrels of pork in 9 months, how many will 500 men consume in the same time?

Ans. $102\frac{4}{7}\frac{4}{3}$ barrels.

34. How much land at 2 dollars 50 cents per acre, must be given in exchange for 360 acres at 3 dollars 75 cents?

Ans. 540 acres.

35. If the earth, which is 360 degrees in circumference, turns round on its axis in 24 hours, how far are the inhabitants at the equator carried in 1 minute, a degree there being $69\frac{1}{2}$ miles? *Ans.* 17 miles 3 fur.

SECTION 2.

SINGLE RULE OF THREE INVERSE.

If in any given question, *more* requires *less*, or *less* requires *more*, the proportion is inverse, and belongs to this rule.

Having stated the question, as in the rule of three direct, proceed according to the following

RULE.

Multiply the first and second terms together, and divide the *product by the* third; the quotient will be the answer, in the *same name* as the second.

EXAMPLE.

1. If 20 men can build a wall in 12 days, how long will it require 40 men to build the same?

M.	*d.*	*M.*
As 20	: 12	:: 40
12		

40)240(6 days. *Ans.*
240

Proof.

d.	*M.*	*d.*
As 6	: 40	:: 12
40		

12)240(20 men.
24

0

2. If 60 men can build a bridge in 100 days, how long will it require 20 men to build it? *Ans.* 300 days.

3. If a wall 100 yards long requires 65 men 4 days, in what time would 5 men complete it? *Ans.* 52 days.

4. If a barrel of flour will last a family of six persons 24 days, how long would it last if 3 more were added to the family? *Ans.* 16 days.

5. If 5 dollars is paid for the carriage of 1*cwt.* weight, 150 miles, how far may 6*cwt.* weight be carried for the same money? *Ans.* 25 miles.

6. If a street 80 feet wide and 300 yards long, can be paved by 40 men in 20 days, what length will one of 60 feet wide be paved by the same men in the same time?

Ans. 400 yards.

7. If a field that is 30 rods wide and 80 in length, contain 15 acres, how wide must one be to contain the same quantity, that is but 70 rods long? *Ans.* 34*R.* 4*ft.* 8$\frac{4}{7}$*in.*

8. If a board be ,75 of a foot wide, what length must it be to measure 12 square feet? *Ans.* 16 feet.

9. How much cloth 1,25 yards wide, can be lined by 42,5 yards of silk that is ,75 of a yard wide?

Ans. 25,5 yards.

10. If 10 men could complete a building in 4,5 months, what time would it require if 5 more were employed?

Ans. 3 months.

11. In what time will 600 dollars gain 50 dollars, when 90 dollars would gain it in 15 years? *Ans.* 2 years.

12. If a traveller can perform a journey in 4 days, when the days are 12 hours long, what time will he require when 16 hours long? *Ans.* 3 days.

400 men in a garrison are supplied with

provisions for 30 days, how many men must be sent out if they would have the provisions last 50 days?

Ans. 160 men.

14. Lent a friend 292 dollars for six months; afterwards I borrow from him 806 dollars; how long may I keep it to balance the favor? *Ans.* 2 months 5 days.

15. 1200 men stationed in a garrison, have provisions for 9 months, at the rate of 14 ounces per day; how long at the same allowance will the same provisions last if they are reinforced by 400 men? And also what diminution must be made on each ration, that the provisions may last for the same time? *Ans.* $6\frac{3}{4}$ mo. at the same allowance—$3\frac{1}{2}$ *oz.* deduction to last for the same time.

16. If a piece of land 40 rods in length and 4 in breadth, make an acre, how wide must it be if it is but 25 rods long?

Ans. $6\frac{2}{5}$ rods.

17. How much in length that is 3 inches broad, will make a square foot? *Ans.* 48 inches.

18. If a pasture field will feed 6 cows 91 days, how long will it feed 21 cows? *Ans.* 26 days.

19. There is a cistern having 1 pipe, which will empty it in 10 hours; how many pipes of the same capacity will empty it in 24 minutes? *Ans.* 25 pipes.

20. How many yards of carpeting that is half a yard wide, will cover a floor that is 30 feet long and 18 feet wide?

Ans. 120 yards.

21. What is the weight of a pea to a steelyard, which being suspended 39 inches from the centre of motion, will equipoise 208*lb.* suspended at the draught end $\frac{3}{4}$ of an inch?

Ans. 4*lb.*

22. A and B depart from the same place, and travel the same road; but A goes 5 days before B at the rate of 20 miles a day, B follows at the rate of 25 miles a day; in what time, and at what distance, will he overtake A?

Ans. 20 days, and 500 miles.

The following rule, if adopted, will suit for the stating of all questions in single proportion, whether *direct* or *inverse*.

GENERAL RULE.

Place that number for the third term, which signifies the same kind, or thing, as that which is sought; and consider whether the number sought will be greater or less; if great-

er, place the least of the other terms for the first, but if less place the greater for the first term, and the remaining one for the second.

Multiply the second and third terms together, and divide the product by the first; the quotient will be the answer required.

EXAMPLES.

1. If 30 horses plow 12 acres, how many will 40 horses plow in the same time?

	h.		*h.*		*acr.*
Direct Proportion.	30	:	40	::	12
			12		

30)480(16 acres. *Ans.*

2. If 30 horses plow 12 acres in 10 days, in how many days will 40 horses plow the same quantity?

	h.		*h.*		*D.*
Inverse Proportion.	40	:	30	::	10
			10		

40)300(7,5 days. *Ans.*

3. If 800 soldiers in a garrison have provisions sufficient for 2 months; how many must depart that the provisions may last them for 5 months? *Ans.* 480.

4. Bought a hogshead of Madeira wine for 119 dollars, nine gallons of which leaked out; what was the remainder sold at per gallon, to gain 12 dollars on the whole?

Ans. 2 dolls. $42\frac{1}{2}$+cts.

5. If 225 pounds be carried 512 miles for 20 dollars, how many pounds may be carried 64 miles for the same money? *Ans.* 1800*lb.*

6. If 87 dolls. 50 cents be assessed on 1750 dolls. what is the tax of 10 dolls. at the same rate? *Ans.* 50 cts.

Promiscuous Questions in Direct and Inverse Proportion.

1. Suppose a man travels to market with his wagon loaded, at the rate of $2\frac{1}{2}$ miles an hour, and returns with it empty at the rate of $3\frac{1}{2}$ miles an hour; how long will he be in performing a journey, going and returning, to a place 123 miles distant? *Ans.* $84\frac{1}{12}$ hours.

2. A lent B 1000 dollars for 189 days, without interest how long should B lend A 650 dollars to requite the favor?
Ans. $290\frac{9}{13}$ days.

3. Bought 14 casks of butter, each weighing 1*cwt.* 1*qr.* 4*lb.* at 12 dollars 60 cents per cwt.; what did they come to, and how much per lb.?
Ans. 226 dolls. 80 cts. whole cost; 11 cts $2\frac{1}{2}$ m. per lb.

4. Sold 4 chests of tea, each weighing 1*cwt.* 0*qr.* 14*lb.* the first for 80 cents per lb. the second for 90 cents, the third for 1 doll. 5 cents, and the fourth for 1 doll. 25 cents; how many pounds of tea were there, what was the average price, and what did the whole come to?
Ans. 504*lb.*—average 1 doll.—come to 504 dolls.

5. When flour is sold at 2 dolls. 24 cents per cwt. what will be the first cost of one dozen of rolls, each weighing 5*oz.* allowing the bread to be in proportion to the flour, as five is to four? *Ans.* 6 cents.

6. If a merchant bought 270 barrels of cider for 780 dollars, and paid for freight 37 dolls. 70 cents, and for other charges and duties 30 dolls. 60 cents; at what must he sell it per barrel to gain 143 dolls. *Ans.* 3 dolls. $67\frac{4}{7}$ cts.

7. If half a ton of hay was equally divided among 80 horses, how much must be given to 7? *Ans.* 3*qr.* 14*lb.*

8. Suppose the circumference of one of the larger wheels of a wagon to be 12 feet, and that of one of the smaller wheels 9 feet 3 inches; in how many miles will the smaller wheel make 1000 revolutions more than the larger?
Ans. 7 m. 5 fur. 34 yds. 1 ft. $7\frac{7}{11}$ in.

9. If a man perform a journey in 18 days, when the days are 15 hours long, how many days will it require to perform the same journey when the days are only 12 hours long? *Ans.* $22\frac{1}{2}$ days.

10. A merchant bought a piece of broadcloth measuring $42\frac{1}{2}$ yds. for 191 dolls. 25 cents; 15 yards of this being damaged, he sells it at two-thirds of its cost; the residue he is willing to sell so as to gain 1 doll. per yard on the whole piece; at what rate must he sell the remainder?
Ans. 6 dolls. $86\frac{4}{11}$ cents per yd.

11. If 60 yards of carpeting will cover a floor that is 30 feet long and 18 broad, what is the width of the carpeting?
Ans. 3 feet.

12. If a piece of land be 40 rods in length, how wide must it be to contain 4 acres? Ans. 16 rods.

13. Suppose a large wheel, in mill work, to contain 70 cogs, and a smaller wheel, working in it, to contain 52 cogs; in how many revolutions of the greater wheel, will the lesser one gain 100 revolutions? *Ans.* 288$\frac{4}{7}$.

14. The number of pulsations in a healthy person is, say 70 in a minute, and the velocity of sound through the air is found to be 1142 feet in a second: now I counted 20 pulsations between the time of observing a flash of lightning from a thunder cloud, and hearing the explosion of the thunder; what was the distance of the cloud?

Ans. 3 m. 5 fur. 145 yds. 2$\frac{1}{2}$ ft.

15. A merchant bought 5 pieces of cloth, of different qualities, but of equal lengths, at the rate of 5, 4, 3, 2, and 1 doll. per yd. for the different pieces; the whole came to 532 dolls. 50 cents; how many yards did each piece contain?

Ans. 35$\frac{1}{2}$ yds.

16. What principal will gain as much in 1 month, as 127 dollars would gain in 12 months? *Ans.* 1524 dolls.

17. If a pair of steelyards be 36 inches in length to the centre of motion, the pea 5 lb. and the draught end $\frac{1}{2}$ inch in length, what weight will they draw? *Ans.* 360 lb.

18. Supposing the above steelyards would only draw 90 lb., what is the length of the draught end?

Ans. 2 inches.

19. If 1 yard of cloth cost 2 dolls. 71 cts. 1$\frac{1}{2}$ mills, what will 67$\frac{1}{2}$ yards come to at the same rate?

Ans. 183 dolls. 4 c. 6$\frac{1}{2}$ m.

20. If a man's income be 16*s.* 5*d.* $1\frac{15}{365}qr.$ per day, what is it per annum? *Ans.* 300*l.*

21. How many pieces of wall paper that is 3 qrs. wide and 11 yards long, will it require to paper the walls of a room that is 25 feet long, 15 wide, and 10$\frac{1}{2}$ high, allowing a reduction of $\frac{1}{10}$ for doors and windows? *Ans.* 10$\frac{2}{11}$.

22. The length of a wall being tried by a measuring line, appears to be 1287 feet 4 inches; but on examination the line is found to be 50 feet 10$\frac{1}{2}$ inches in length, instead of 50 feet its supposed length; required the true length of the wall? *Ans.* 1309 feet 10$\frac{17}{50}$ inch.

23. If a dealer in liquors use, instead of a gallon, a measure which is deficient by half a pint, what will be the true measure of 100 of these false gallons? *Ans.* 93$\frac{3}{4}$ galls.

SECTION 3.

THE DOUBLE RULE OF THREE.

THE DOUBLE RULE OF THREE, or as it is often called, Compound Proportion, is used for solving such questions as have five terms given to find a sixth. In all questions belonging to this rule, the three first terms must be a ***supposition***, the two last a *demand*.

RULE FOR STATING.

1. Set the two terms of the supposition, one under the other.

2. Place the term of the same kind with the answer sought in the second place.

3. Set the terms of the demand, in the third place, observing to place the correspondent terms of the supposition and demand in the same line. Consider the upper and lower extremes, with the middle terms, separately, as in the single rule of three; if both lines are *direct*, then the question will be in direct proportion; but if either lines are *inverse*, then will the question be in inverse proportion.

When the question is in direct proportion, multiply the product of the two last terms by the middle term for a dividend, and multiply the two first terms for a divisor; the quotient will be the answer in the same name with the middle term.

But if the proportion be inverse, transpose the inverse terms and proceed in the same manner as in direct proportion.

DIRECT PROPORTION.

EXAMPLE.

1. If 6 men in 8 days earn 100 dollars, how much will 12 men earn in 24 days?

		dolls.		
6 men 8 days	:	100	::	12 men 24 days
48				288
				100

48) 28800 (600 dolls. *Ans.*
288
00

2. If 10 bushels of oats suffice 15 horses for 20 days, how many bushels will serve 60 horses 36 days?

Ans. 60 bushels.

3. If 56 pounds of bread will suffice 7 men 14 days, how much bread will serve 21 men 3 days? *Ans.* 36 pounds.

4. If 8 students spend 384 dollars in 6 months, how much will maintain 12 students 10 months? *Ans.* 960 dollars.

5. If 20 hundred weight is carried 50 miles for 25 dollars, how much must be given for the carriage of 40 hundred weight 100 miles? *Ans.* 100 dollars.

6. If 14 dollars interest is gained by 700 dollars in 6 months, what will be the interest of 400 dollars for 5 years?

Ans. 80 dollars.

7. If 4 men can do 12 rods of ditching in 6 days, how many rods may be done by 8 men in 24 days?

Ans. 96 rods.

INVERSE PROPORTION.

EXAMPLE.

1. If 4 dollars pay 8 men for 3 days, how many days must 20 men work for 40 dollars?

		days.		
As 4 dolls.	}	: 3	::	} 40 dolls.
8 men.	}			} 20 men.

Here the lower line is inverse, which transposed will stand thus:

		days.		
As 4 dolls.	}	: 3	::	} 40 dolls.
20 men.	}			} 8 men.

80 — 320 — 3

80) 960 (12 days. *Ans.*
80

160
160

2. If 4 men are paid 24 dollars for 3 days work, how many days may 16 men be employed for 384 dollars?

Ans. 12 days.

3. If 4 men are paid 24 dollars for 3 days work, how many men may be employed 16 days for 96 dollars?

Ans. 3 men.

4. If 7 men can reap 84 acres of grain in 12 days, how many men can reap 100 acres in 5 days? *Ans.* 20 men.

5. If 7 men can reap 84 acres of grain in 12 days, how many days will it require 20 men to reap 100 acres?

Ans. 5 days.

6. If 40 cents are paid for the carriage of 200 pounds for 40 miles, how far may 20200 pounds be carried for 60 dollars 60 cents? *Ans.* 60 miles.

7. If 5 men spend 200 dollars in 22 weeks and 6 days, how long will 300 dollars support 12 men?

Ans. 14 weeks 2 days.

Promiscuous Questions.

1. If 12 oxen in 8 days eat 10 acres of clover, how many acres will serve 24 oxen 48 days? *Ans.* 120 acres.

2. A person having engaged to remove 8000 weight 15 miles in 9 days; with 18 horses, in 6 days, he removed 4500 weight; how many horses will be necessary to remove the rest, in the remaining 3 days? *Ans.* 28 horses.

3. If the carriage of 9 hogsheads of sugar, each weighing 12 cwt., for 60 miles, cost 100 dollars, what must be paid for the carriage of 50 barrels of sugar, each weighing 2,5 cwt., 300 miles? *Ans.* 578 dolls. 70+cents.

4. If 1 pound of thread make 3 yards of linen, 5 quarters wide, how many pounds of thread will it require to make a piece of linen 45 yards long and 1 yard wide?

Ans. 12 lb.

5. If a footman travels 240 miles in 12 days, when the days are 12 hours long; in how many days will he travel 720 miles, when the days are 16 hours long?

Ans. 27 days.

6. A perch of stone measures 16½ feet long, 1½ foot broad, and 1 foot high; at 1 doll. 25 cts. per perch, what will a pile of stone come to, which measures 30 feet long, 26 feet broad, and 4½ feet high? *Ans.* 177 dolls. 27+cts.

7. How many cords are there in a pile of wood 200 feet long, 10 feet high, and 36 feet broad; the cord measuring, according to law, 8 feet in length, 4 in breadth, and 4 in height? *Ans.* 562½ cords.

8. If 3 pounds of cotton make 10 yards of cloth, 6 qr. wide, how many pounds will it take to make a piece 10 yards long and 3 qr. wide? *Ans.* 15 lb.

9. If 24 men build a wall 200 feet long, 8 feet high, and 6 feet thick, in 80 days, in what time will 6 men build one 20 ft. long, 6 ft. high, and 4 ft. thick? *Ans.* 16 days.

10. If a family of 9 persons spend 450 dolls. in 5 months, how much would they spend in 8 months, if 5 more were added to the family? *Ans.* 1120 dollars.

11. If a baker's bill for a family of 8 persons amounts to 11¼ dolls. in a month, when flour is at 10 dolls. per barrel, what will it amount to in 6 months if 4 more are added to the family, and flour is at 11 dollars per barrel?

Ans. 111 dolls. 37½ cts.

12. If a cellar which is 22,5 feet long, 17,3 feet wide, and 10,25 deep, is dug by 6 men in 2,5 days, working 12,3 hours each day; how many days of 8,2 hours will it require 9 men to dig one which is 45 feet long, 34,6 wide, and 12,3 feet deep? *Ans.* 12 days.

PART V.

MERCANTILE ARITHMETIC.

SECTION 1.

OF PRACTICE.

PRACTICE, so called from its frequent use in business, is only a contraction of the preceding rules of proportion. By it a compendious way is given of finding the price of any given quantity of goods or other articles of trade, when the price of 1 is known.

Case 1.

When the price consists of dollars, cents, and mills.—Reduce the given quantity by multiplication, as in whole numbers, and point off from the right of the product for mills and cents, according to the rules in federal money. Or, multiply by the dollars only, and take aliquot or fractional parts for the cents and mills.

TABLE.

50 cents is	$\frac{1}{2}$ of a dollar.	$14\frac{2}{7}$ cents is	$\frac{1}{7}$
$33\frac{1}{3}$.	$\frac{1}{3}$ . .	$12\frac{1}{2}$.	$\frac{1}{8}$
25 .	$\frac{1}{4}$ . .	$11\frac{1}{9}$.	$\frac{1}{9}$
20 .	$\frac{1}{5}$.	10 .	$\frac{1}{10}$
$16\frac{2}{3}$.	$\frac{1}{6}$.	5 .	$\frac{1}{20}$

EXAMPLE.

1. What will 175 pounds of tea come to at 1 dollar 30 cents and 5 mills per lb.?

130,5	cts.			175
175	Or, 25		$\frac{1}{4}$	4375
	5		$\frac{1}{5}$	875
6525	5 m.		$\frac{1}{10}$	875
9135				
1305				228375
228375				

Ans. 228 dolls. 37 cts. 5 mills.

		D. c. m.	*D. c. m.*
2.	250 yards at	1,75	*Ans.* 437,50
3.	201 do.	4,20	884,20
4.	2210 do.	1,10	2431,00
5.	421 do.	2,41,5	1016,71,5
6.	625 do.	25	156,25
7.	8275 do.	4,4	364,10
8.	8275 do.	5	41,37,5

Case 2.

When the price is the fractional part of a dollar, or cent, such as $\frac{3}{4}$ of a dollar, $\frac{3}{4}$ of a cent, multiply the quantity by the numerator, and divide the product by the denominator; the quotient will be the answer.

EXAMPLE.

1. What will 375 yards of muslin cost at $\frac{3}{4}$ of a dollar per yard?

375 at $\frac{3}{4}$

3

4)1125

281,25 *Ans.*

	cwt.	*qr.*	*lb.*		*D. c. m.*
2.	4	1	14	of sugar at $\frac{1}{4}$ of a doll. per *lb.*	*Ans.*
3.	12	2	13	of spice at $\frac{3}{8}$ do.	do.
4.	14	3	7	of lead at $\frac{1}{7}$ do.	for 5lb.

Application.

1. Bought 6 hogsheads of tobacco, each weighing 12,5 cwt. at $\frac{3}{8}$ of a doll. per pound; what did it cost?

Ans. 3150 dolls.

2. A gentleman bought a vessel of 60 tons burden, and gave at the rate of $2\frac{3}{5}$ eagles per ton; what did the vessel cost? *Ans.* 1560 dolls.

3. A carpenter bought 12650 feet of boards at $10\frac{3}{4}$ dollars per thousand; what did they cost him?

Ans. 137 dolls. 56 cts. $8\frac{1}{2}$ m.

Case 3.

When the price and quantity given are of several denominations, multiply the price by the integers, or whole numbers, and take aliquot parts for the rest.

Table of a hundred weight.

56 lb.	is		of a Cwt.	14 lb.	is		of a Cwt.
28	.	$\frac{1}{2}$	.	8	.	$\frac{1}{7}$	.
16	.	$\frac{1}{4}$	.	7	.	$\frac{1}{8}$	.

EXAMPLE.

1. Bought 16*cwt.* 1*qr.* 16*lb.* of tobacco at 12 dollars 44 cents per hundred weight; what did it cost?

12,44
16

74[6]4
1244

1*qr.* is	$\frac{1}{4}$	19904
16*lb.*	$\frac{1}{4}$	311
		1777$\frac{1}{4}$

2039271$\frac{1}{4}$ *Ans.* 203 dolls. 92 cts. 7$\frac{1}{4}$ ms.

2. 17*cwt.* 3*qr.* 19*lb.* of sugar at 10 dollars 94 cents per hundred weight. *Ans.* 196 dolls. 4cts.

3. 5*cwt.* 1*qr.* 0*lb.* of tobacco at 13 dollars 41 cents per hundred weight. *Ans.* 70 dolls. 40 cts. 2 m.

4. 7*cwt.* 0*qr.* 19*lb.* of sugar at 15 dolls. 5 mills per hundred weight. *Ans.* 107 dolls. 58 cts.

Case 4.

When the price consists of pounds, shillings, pence, and farthings.

1. Reduce the given price to dollars and cents, (see reduction of money, page 41) and then proceed according to the foregoing cases. Or,

2. Multiply by the integers, and take aliquot parts for the remainder.

TABLE.

s.	*d.*			*d.*		
10		is	$\frac{1}{2}$ of a pound.	6	is	$\frac{1}{2}$ of a shilling.
6	8	—	$\frac{1}{3}$	4	—	$\frac{1}{3}$
5		—	$\frac{1}{4}$	3	—	$\frac{1}{4}$
4		—	$\frac{1}{5}$	2	—	$\frac{1}{6}$
3	4	—	$\frac{1}{6}$			
2	6	—	$\frac{1}{8}$			
2		—	$\frac{1}{10}$			
1	8	—	$\frac{1}{12}$			

EXAMPLES.

1. What will 4548 yards come to at 1*s.* 6*d.* per yard?

1*s.* 6*d.*=18*d.*=20 cents=$\frac{1}{5}$ of a dollar.

cts.		
20	$\frac{1}{5}$	4548

909,60 dollars. *Ans.*

Or, 6*d.* | $\frac{1}{2}$ | 4548 at 1 shilling, will be the same number of shillings.
2274

2|0)682,2 shillings

£.341 2=909 dolls. 60 cents.

		s.	*d.*		£.	*s.*	*d.*	*D.*	*c.*
2.	473 yards at	6	8	*Ans.*	157	13	4=	420,	44$\frac{4}{9}$
3.	397 do.	3	4		66	3	4=		
4.	159$\frac{1}{4}$*lb.* of coffee at	1	8		13	5	5=		
5.	658*lb.* of tea at	12			394	16	0=		
6.	745 yds. of cloth at	16			596	0	0=		
7.	969 do.	19	11		964	19	3=		
8.	3715 do.	9	4$\frac{1}{2}$		1741	8	1$\frac{1}{2}$=		
9.	4567 do.	19	11$\frac{1}{2}$		4557	9	8$\frac{1}{2}$=		

Case 5.

When both the price of the integer and the quantity are of different denominations, multiply the price by the integers of the quantity, and take parts of the price for those of the integer.

EXAMPLES.

1. What will 45*cwt.* 2*qr.* 14*lb.* of sugar come to at 3*l.* 7*s.* 9*d.* per hundred weight?

						£.	s.	d.	
5*s.* is	¼	45	2	14					
2	1/10	3	7	9	Or thus,	3	7	9	
									5×9=45
		135							
		11	5			16	18	9	
6*d.* is	¼	4	10					9	
3	½	1	2	6					
			11	3		152	8	9	
2*qr.* is	½	1	13	10½	2*qr.* is ½	1	13	10½	
14*lb.*	¼		8	5½	14*lb.* ¼		8	5½	
		£.154	11	1 *Ans.*		£.154	11	1	

2. 37*T.* 14*cwt.* 2*qr.* 14*lb.* of hemp at 89*l.* 6*s.* 8*d.* per ton. *Ans.* 3370*l.* 13*s.* 2*d.*

3. 3*T.* 12*cwt.* 3*qr.* 27*lb.* of sugar at 5*l.* 11*s.* 5*d.* per cwt. *Ans.* 625*l.* 11*s.* 10*d.*

4. 7*T.* 4*cwt.* 2*qr.* 21*lb.* of rice at 3*l.* 17*s.* 6*d.* per cwt. *Ans.* 560*l.* 13*s.* 3¼*d.*

5. 476 acres 3 roods 28 perches of land at 3*l.* 7*s.* 11*d.* per acre. *Ans.* 1619*l.* 11*s.* 1¾*d.*

6. 640 acres 2 roods 20 perches at 10 dollars 55 cents per acre. *Ans.* 6758 dolls. 59½ cts.

7. 229 acres 3 roods 18 perches at 18 dollars 50 cents per acre. *Ans.* 4252 dolls. 45½ cts.

8. 12*cwt.* 0*qr.* 7*lb.* at 6 dollars 34 cents per cwt. *Ans.* 76 dolls. 47 cts. 6 m.

9. 17*cwt.* 3*qr.* 24¼*lb.* at 14 dollars per cwt. *Ans.* 251 dolls. 56 cts. 2½ m.

10. 16 acres 3 roods 25 perches, at 125 dollars 50 cents per acre. *Ans.* 2121 dolls. 73 cts. 3+m.

11. 25*cwt.* 3*qr.* 14*lb.* at 3*l.* 17*s.* 6*d.* per cwt. *Ans.* 100*l.* 5*s.* 3¾*d.*

SECTION 2.

Allowance on the weight of goods, called Tare and Tret.

Tare, is an allowance made for the weight of the barrel, box, trunk, &c. in which goods are packed.

Tret, is an allowance to retailers, for waste in the sales of their commodities.

Gross, is the whole weight of the goods with the barrel, box, &c. in which they are put up.

Neat, is the weight of the goods after all allowances are deducted.

RULE.

Find the amount of the tare, and subtract it from the gross weight, the remainder will be the neat.

EXAMPLE.

1. What is the neat weight of a hogshead of tobacco, weighing gross 12*cwt.* 3*qr.* 12*lb.* tare 14*lb.* per cwt.

lb.		*cwt.*	*qr.*	*lb.*	
14		12	3	12	gross
12		1	2	12	tare
168	= the tare of 12*cwt.*	*Ans.* 11	1	0	neat
10 8*oz.*	= the tare of 3*qr.*				
1 8*oz.*	= the tare of 12*lb.*				
180	= 1*cwt.* 2*qr.* 12*lb.*				

2. What will 3 barrels of sugar come to, weighing as follows: *viz.* No. 1, 2*cwt.* 1*qr.* 25*lb.* No. 2, 2*cwt.* 2*qr.* No. 3, 2*cwt.* 21*lb.* tare 21*lb.* per barrel; at 12 dollars 50 cents per cwt.? *Ans.* 82 dolls. 47 cts. 7 m.

3. At 45 cents per pound, what will 4 barrels of indigo come to, weighing as follows:

	cwt.	*qr.*	*lb.*		*lb.*
No. 1,—	3	3	2	Tare	29
No. 2,—	4	1	10	—	36
No. 3,—	4	0	19	—	32
No. 4,—	4	0	0	—	35

Ans. 760 dolls. 95 cts.

4. Bought 2 hogsheads of sugar, weighing as follows *viz.* No. 1, 11*cwt.* 1*qr.* 17*lb.* tare 112*lb.* No. 2, 12*cwt.* 2*qr* tare 74*lb.* at 16 dollars 80 cents per cwt. neat; for which I gave 18 barrels of flour at 4 dollars 50 cents per bbl. and 1½ ton of iron at 120 dollars per ton; what was the balance still due? *Ans.* 112 dolls. 65 cts.

5. What is the neat weight of 12 barrels of potash, each weighing 4*cwt.* 2*qr.* 26*lb.* tare 12*lb.* per cwt.; and what will it come to at 9 dollars per cwt.?

Ans. 50*cwt.* 2*qr.* 23*lb.* and comes to 456 dolls. 34¾ cts.

6. Sold a hogshead of sugar, weighing 6*cwt.* gross, tare 100*lb.* tret 4*lb.* per 104, for 82 dolls. 50 cents; what was it sold for per pound? *Ans.* 15 cents.

7. In 120*cwt.* 3*qr.* gross, whole tare 177*lb.* tret 4*lb.* per 104, how much is the neat weight in pounds, and what the amount at 73 cents per pound?

Ans. 12833,6*lb.* and comes to 9368 dolls. 53 cts.

8. Bought 9 hogsheads of sugar, each weighing 6*cwt.* 2*qr.* 12*lb* gross; tare 17*lb.* per cwt. what is the neat weight. and what does it amount to at 16 dollars per cwt.?

Ans. 50*cwt.* 1*qr.* 22*lb.* amounts to 807 dolls. 14½ cts.

9. Sold 27 bags of coffee, each 2*cwt.* 3*qr.* 17*lb.* gross; tare 13*lb.* per cwt. tret 4*lb.* per 104; what is the neat weight, and what will it come to at 32 cents per pound?

Ans. 66*cwt* 2*qr.* 11*lb.* and comes to 2386 dolls. 88 cts.

SECTION 3.

OF INTEREST.

INTEREST is a compensation allowed for the use of money for a given time; and is generally throughout the United States fixed by law at the rate of 6 dollars for every 100, per annum.

1. The sum of money at interest, is called the *Principal.*

2. The sum per cent. agreed on, is called the *Rate.*

3. The principal and interest added together, is called the *Amount.*

Interest is either *simple* or *compound.*

SIMPLE INTEREST.

Simple Interest is a compensation arising from the principal only.

Case 1.

When the given time is one or more years, and the principal dollars only.

RULE.

Multiply the given sum by the rate per cent.; the product will be the interest for one year in cents, which multiplied by the number of years, will be the answer required.

EXAMPLES.

1. What is the interest of 454 dollars for one year, at 6 per cent.?

454
6

2724 cents. *Ans.* 27 dolls. 24 cts.

2. Required the interest of the same sum for 5 years, at the same rate?

454
6

2724
5

13620 cents. *Ans.* 136 dolls. 20 cts.

3. Required the amount of the same sum for 5 years, at he same rate?

454
6

2724
5

13620 interest.
45400 principal.

59020 amount. *Ans.* 590 dolls. 20 cts.

4. What is the interest of 200 dollars for 2 years, at 6 per cent? Ans. 24 dolls.

5. What is the interest of 1260 dolls. for 4 years, at 7 per cent.? *Ans.* 352 dolls. 80 cts.

6. What is the amount of a note for 560 dollars for 3 years, at 8 per cent.? *Ans.* 694 dolls. 40 cts.

7. What sum must be given to discharge a bond given for 4520 dollars, on which there is 6 years interest at 5 per cent.? *Ans.* 5876 dolls.

Note. When the rate per cent. contains a fraction, such as $\frac{1}{4}$, $\frac{1}{2}$, $\frac{3}{4}$ the principal must be multiplied by the fraction, as well as the whole number: this may be done either by adding the parts of $\frac{1}{4}$, $\frac{1}{2}$, &c. of the principal to the product of the whole number; or reduce the fraction to a decimal. *See case* 1, *in reduction of decimals.*

8. What is the amount of 400 dollars for 2 years, at $6\frac{1}{2}$ per cent.? *Ans.* 452 dolls.

9. What is the interest of 4925 dollars for 9 years, at $\frac{3}{4}$ per cent.? *Ans.* 3324 dolls. 37 cts. 5 m.

10. What is the amount of 2500 dollars for 1 year, at $7\frac{1}{2}$ per cent.? *Ans.* 2693 dolls. 75 cts.

Case 2.

When the principal is dollars and cents, or dollars, cents, and mills, and the time years only.

RULE.

Multiply the given sum by the rate per cent. and divide the product by 100; or, what is the same, point off two figures on the right of the product; the quotient, or remaining figures, will be the answer, in the same name with the lowest denomination in the principal.

EXAMPLES.

1. What is the interest of 264 dollars 50 cents for 1 year, at 6 per cent.?

264,50
6

cents 1587,00 *Ans.* 15 dolls. 87 cts.

2. What is the interest of 468 dollars 22 cents and 5 mills for 1 year, at 8 per cent.

468,225
8

mills 37458,00 *Ans.* 37 dolls. 45 cts. 8 m.

3. What is the interest of 364 dollars 50 cents for 5 years, at 6 per cent. per annum?

36450	Or, 36450	
6		30=product of the rate
		and time.
218700	cents 10935,00	
5		

cents 10935,00 *Ans.* 109 dollars 35 cents.

4. What is the amount of a note for 1260 dollars 50 cents and 5 mills for 3 years, at 7½ per cent. per annum?

Ans. 1544 dolls. 11 cts. 8+ms.

5. What sum will discharge a bond given for 630 dollars 50 cents, on which there is 5 years interest at 8 per cent. per annum? *Ans.* 882 dolls. 70 cts.

6. What is the difference between the interest of 1274 dollars 64 cents 6 mills for 3 years, at 7½ per cent. per annum, and the interest of 3462 dollars 84 cents, for 4 years, at 3¼ per cent. per annum?

Ans. The latter is 163 dolls. 37 c. 3,85 m. the greater.

7. A gave B his bond for 3422 dolls. 25 cents, to be paid in the following manner, viz. one-third at the end of one year, one-third at the end of two years, and the remainder at the end of three years, with interest from the date, at 6 per cent. per annum; what will be the annual payments, and what the whole amount?

Ans. 1st payment 1209 dolls. 19,5 cts. 2d, 1277 dolls. 64 cts. 3d, 1346 dolls. 8,5 c.; whole amount 3832 d. 92 c.

Case 3.

When the principal is dollars, cents, &c. and the time is years and months, or months only.

RULE.

Multiply by half the number of months in the given time, when the rate is 6 per cent. per annum: but if the rate per cent. be more or less than 6 per cent., multiply the given number of months by the rate, and divide the product by 12; the quotient will be the rate for the time; the principal multiplied by this rate, will give the interest required.

EXAMPLES.

1. What is the interest of 650 dollars for 8 months per cent. per annum?

650
4 half the months

cents 2600 *Ans.* 26 dollars.

2. What is the interest of 860 dollars for 1 year months, at 6 per cent. per annum?

860
9 half the months

cents 7740 *Ans.* 77 dolls. 40 cents

3. What is the interest of 420 dollars for 9 months per cent. per annum?

9 months
8 per cent.

12)72

6 the rate for the time.

420
6

2520 *Ans.* 25 dolls

4. What is the amount of a note for 724 dollars, wi months interest due thereon, at 4 per cent. per annum? *Ans.* 767 dolls. 44

5. What is the interest of 240 dollars for 15 mont $7\frac{1}{2}$ per cent. per annum? *Ans.* 22 dolls. 50

6. What is the interest of 1260 dollars for 4 mont $6\frac{1}{2}$ per cent. per annum? *Ans.* 27 dolls. 30

Case 4.

When the principal is dollars, cents, &c. and the ti months and days, or days only.

RULE.

Find the interest for the given months by the last and take aliquot parts for the days.

Note. In calculation of interest, 30 days make a month.

Or, multiply the given sum when the rate is 6 per by the number of days, and divide the product by 60 tient is the interest required.

Both the foregoing methods are considered suffic n business, by merchants and accomptants gene

yet as this is only allowing 360 days in the year, and not 365, the true time; if, therefore, the principal is large, on which interest is due, and greater exactness is required, then find the interest of the given sum for 1 year, and proceed according to the single rule of three. As 365 days : to the interest for one year :: the given number of days : the answer. Or by the double rule of three, find the fixed divisors, which for 5 per cent. is 7300, for 6 per cent. is 6083, for 7 per cent. 5214; multiply the principal by the days, and divide by these divisors according to the rate per cent. required.

EXAMPLES.

1. What is the interest of 260 dollars for 5 months and 20 days, at 6 per cent. per annum?

260
2

1 month	$\frac{1}{4}$	5,20	interest for 4 months
15 days	$\frac{1}{2}$	1,30	interest for 1 do.
5	$\frac{1}{3}$	65	interest for 15 days
		21,6	interest for 5 do.

7,36,6 *Ans.* 7 dolls. 36 cts. 6 m.

2. What is the interest of 450 dollars for 36 days, at 6 per cent. per annum?

450
36

2700
1350

16200

6,0) 1620,0

2,70 *Ans.* 2 dolls. 70 cts.

3. What is the interest of 564 dollars for 44 days, at 6 per cent. per annum?

564
6

	days			days
As	365	:	3384	:: 44

44

13536
13536

365) 148896 (4,079, *Ans.* 4 dolls. 07 cts. 9 + m.
1460

2896
2555

3410
3285

125

4. What is the interest of 960 dollars for 70 days, at 6 per cent. per annum?

960
70

6083) 67200 (11 + dolls. *Ans.*
6083

6370
6083

287

5. What is the interest of 12000 dollars for 40 days, at 7 per cent. per annum? *Ans.* 92 dolls. 6 cts.

6. What is the interest of 8400 dollars for 20 days, at 5 per cent. per annum? *Ans.* 23 dolls.

	D.	*c.*	*days*						*D.c.m.*
7.	517,	90	for 84	at 6	per	cent.	per	annum.	*Ans.* 7,15,1
8.	73,	41	27	-	-	-	-	-	33
9.	225,	24	40	-	-	-	-	-	1,48,1
10.	1200,	00	80	-	-	-	-	-	15,78,1
11.	2962,	19	254	-	-	-	-	-	123,68,8
12.	1733,	97	102	-	-	-	-	-	29,07,5

A TABLE,

Showing the number of days, from any day in any month, to the same day in any other month through the year.

From	Ja.	Fb.	Mr.	Ap.	Ma.	Ju.	Jly.	Au.	Se.	Oc.	No.	De.
To Jan.	365	334	306	275	245	214	184	153	122	92	61	31
Feb.	31	365	337	306	276	245	215	184	153	123	92	62
March	59	28	365	334	304	273	243	212	181	151	120	90
April	90	59	31	365	335	304	274	243	212	182	151	121
May	120	89	61	30	365	335	304	273	242	212	181	151
June	151	120	92	61	31	365	335	304	273	243	212	182
July	181	150	122	91	61	30	365	334	303	273	242	212
August	212	181	153	122	92	61	31	365	334	304	273	243
Sept.	243	212	184	153	123	92	62	31	365	335	304	274
Oct.	273	242	214	183	153	122	92	61	30	365	334	304
Nov.	304	273	245	214	184	153	123	92	61	31	365	335
Dec.	334	303	275	244	214	183	153	122	91	61	30	365

Suppose the number of days between the 10th of April October were required; under the column of

April at the top of the table, look for October, and you find 183, the number required.

If the days in the given months be different, their difference must be added or subtracted, to or from the tabular number. Thus, from the 10th of April to the 20th of October, is 183+10=193 days. And from the 20th of April to the 10th of October, 183—10=173 days.

If the time exceed a year, 365 days must be added for each year.

Case 5.

When the amount, rate, and time are given to find the principal.

RULE.

As the amount of 100 dollars, at the rate and time given, is to 100 dollars, so is the amount given to the principal required.

EXAMPLE.

1. What principal being put to interest for 9 years, at 5 per cent. per annum, will amount to 725 dollars?

9
5
—
45
100
—

As 145 : 100 :: 725 : 500 *Ans.*

2. What principal being put to interest for 12 years, at 6 per cent. per annum, will amount to 2752 dollars?

Ans. 1600 dolls.

3. Received 728 dollars as payment in full for a note with 5 years interest thereon, at 6 per cent. per annum; for how much was the note given? *Ans.* 560 dolls.

4. What sum put to interest for 4 years, at 7½ per cent. per annum, will amount to 1638 dollars?

Ans. 1260 dolls.

5. Received 2000 dollars as payment in full for a bond, with 5 years interest thereon, at 5¾ per cent. per annum; what principal did the bond contain?

Ans. 1553 dolls. 39 cts. $8\frac{1}{23}$.

Case 6.

When the amount, time, and principal are given to find the rate.

RULE.

1. As the principal is to the interest for the whole time so is 100 dollars to its interest for the same time.

2. Divide the interest so found by the time, and the quotient will give the rate per cent.

EXAMPLE.

1. At what rate of interest per cent. will 500 dollars amount to 725 dollars in 9 years?

725
500
—
225

	D.		D.		D.		D.
As	500	:	225	::	100	:	45

9)45
—
5 per cent. *Ans.*

2. Paid 858 dollars in full for a note given for 650 dollars, with 4 years interest due thereon; what was the rate per cent. per annum charged on said note? *Ans.* 8 per cent.

3. At what rate per cent. will 1600 dollars amount to 2752 dollars in 12 years? *Ans.* 6 per cent.

4. At what rate per cent. will 640 dollars amount to 860 dolls. 80 cents in 6 years? *Ans.* $5\frac{3}{4}$ per cent.

5. At what rate per cent. will 12000 dollars amount to 20100 dollars in 15 years? *Ans.* $4\frac{1}{2}$ per cent.

Case 7.

When the principal, amount, and rate are given to find the time.

RULE.

Find the interest of the principal for one year. And then, as the interest for 1 year, is to 1 year, so is the whole interest to the time required.

EXAMPLE.

1. In what time will 500 dollars amount to 725 dollars at 5 per cent. per annum?

500
5
—

			D.		Y.		D.		Y.	
Interest for 1 year	25,00	As	25	:	1	::	225	:	9	*Ans.*
Amount	725									
Principal	500									
Whole interest	225									

2. In what time will 650 dollars amount to 910 dollars, at 8 per cent. per annum? *Ans.* 5 years.

3. In what time will 1600 dollars amount to 2080 dollars at 6 per cent. per annum? *Ans.* 5 years.

Case 8.

When the principal is in English money, *viz.* pounds, shillings, and pence, and the interest required either in Federal or English money.

RULE.

Reduce the English to Federal money, and find the interest by the preceding rules.

EXAMPLE.

1. What is the interest in Federal money, of 325*l.* 10*s.* English money, for 5 years, at 6 per cent. per annum?

£.	*s.*	*D.*	*c.*	
325	10	= 1445	22	
		30		= the time multiplied by the rate.
4335660				*Ans.* 433 dolls. 56 cts. 6m

2. What is the amount of a note for 640*l.* 3*s.* 6*d.* with 3 years interest due thereon, at 5 per cent. per annum, in Federal money? *Ans.* 3268 dolls. 73 cts. $3\frac{1}{2}\frac{1}{6}$ m.

3. What is the interest of 1374*l.* 1*s.* 9*d.* for $1\frac{1}{2}$ year, at $5\frac{1}{8}$ per cent. per annum? *Ans.* 115*l.* 18*s.* 9*d.*

Case 9.

Computing interest on bonds, notes, &c. on which different payments have been made.

RULE I.

Find the interest of the principal from the time the interest first commenced, to the time of the first payment made; add that interest to the principal, and subtract from the amount the payment made; the remainder forms a new principal; on which proceed in the same manner, till all the payments are brought in.

Note 1. When a payment alone, or in conjunction with any preceding payment, is less than the interest due at the time, then no calculation must be made; but these lesser payments added to the next.

2. By this rule, no part of the interest ever forms a part of the principal carrying interest, the payments being first applied to discharge the interest.

EXAMPLES.

1. A has B's note for 1000 dollars, dated 1st January 1816, payable in 18 months, with interest from the date, at 6 per cent. per annum. On which the following payments are endorsed, *viz.*

1816. July	1.	Rec'd on the within note	230 dollars
1817. Jan.	1.	Rec'd - - -	300
March	1.	Rec'd - - -	4
April	1.	Rec'd - - -	250

What was the balance due on the 1st of July, 1817, when the whole note is payable?

	D.	*c.*
Principal at interest from January 1, 1816,	1000	00
1816, July 1. Interest (6 months) - -	30	
	1030	
Paid same date - - - - - -	230	
Remainder for a new principal - -	800	
1817, January 1. Interest (6 months) -	24	
	824	
Paid same date - - - - -	300	
Remainder for a new principal - -	524	
March 1. Paid 4 dollars less than the interest and not to be calculated.		
April 1. Interest (3 months) - - -	7	86
	531	86
Paid same date 250+4= - - -	254	
	277	86
July 1. Interest (3 months) - - -	4	17
Balance due - - - *Ans.*	282	03

RULE II.

Multiply the principal by the number of days, till the first payment is made; the remaining principal by the number of days, between the first and second payment, &c. till all the payments are made; divide the whole amount by 60; the quotient will give the interest required.

Note. By this method the interest is generally calculated among merchants.

1. A bond was given by B to C, for 2400 dollars, payable in 2 years, with interest from the date. Dated July 1, 1815. On this bond the following payments are endorsed; *viz.* May 1, 1816, 900 dollars; October 1, 1816, 450 dollars, January 1, 1817, 620 dollars. Required the amount due on the 1st of May, 1817?

1815. July 1. Principal 2400 dollars.

1816. May 1.	2400	multiplied by	304	is	729600
Paid	900				
Oct. 1.	1500	. . . .	153	.	229500
Paid	450				
1817. Jan. 1.	1050	. . . .	92	.	96600
Paid	620				
May 1.	430	. . . .	120	.	51600
Interest	184 55				
		Divide by 6,0			110730,0
Balance	614 55	*Ans.*			
				Interest	184,55

2. A note was given by A to B, for 1800 dollars, dated 1st January, 1820, with interest from the date. On which the following payments are endorsed, *viz.* April 1, 1821, 700 dollars; January 1, 1822, 400 dollars; July 1, 1822, 500 dollars. Required the amount due on the 1st of January, 1823? *Ans.* 414 dolls. 16 cts. 6 m.

COMPOUND INTEREST

Is a compensation allowed not only for the principal, but also for the interest as it becomes due.

RULE.

Add the simple interest of the given sum for one year to the principal. This amount forms a new principal for the second year, and so on for any number of years required.

Subtract the first principal from the last amount, the remainder will be the compound interest required.

EXAMPLE.

1. What is the compound interest of 500 dollars for 3 years, at 6 per cent. per annum?

dolls.	
500	1st principal
6	
30,00	interest
500	
530,00	2d principal
6	
31,80,00	
530	
561,80	3d principal
6	
33,70,80	
561,80	
595,50,8	last amount
500	
95,50,8	*Ans.* 95 dolls. 50 cents, 8 mills.

Compound Interest may be more expeditiously calculated by the following Table, in which the amount of one dollar for any number of years under 30 *is shown, at the rates of* 5 *and* 6 *per cent. per annum, compound interest.*

Years	Rates 5	Rates 6	Years	Rates 5	Rates 6
1	1.05000	1.06000	16	2.18287	2.54035
2	1.10250	1.12360	17	2.29201	2.69277
3	1.15762	1.19101	18	2.40662	2.85434
4	1.21550	1.26247	19	2.52695	3.02559
5	1.27628	1.33822	20	2.65329	3.20715
6	1.34009	1.41852	21	2.78596	3.39956
7	1.40710	1.50363	22	2.92526	3.60353
8	1.47745	1.59384	23	3.07152	3.81975
9	1.55132	1.68948	24	3.22510	4.04893
10	1.62889	1.79084	25	3.38635	4.29187
11	1.71034	1.89829	26	3.55567	4.54938
12	1.79585	2.01219	27	3.73345	4.82234
13	1.88565	2.13292	28	3.92013	5.11168
14	1.97993	2.26090	29	4.11613	5.41838
15	2.07892	2.39655	30	4.32194	5.74349

— d the compound interest of any sum by this table, e figures opposite the number of years, under the nt. by the given principal; the product will be required; from this subtract the principal, the ill be the interest.

EXAMPLE.

2. What is the compound interest of 1000 dollars for 8 years, at 6 per cent. per annum?

1,59384 the tabular number for the time
1,000 the principal

1593,84000
1000

593,84 the interest. *Ans.* 593 dolls. 84 cts.

3. What is the amount of 1500 dollars for 5 years, at 5 per cent. per annum? *Ans.* 1914 dolls. 42 cts.

4. What is the compound interest of 4500 dollars for 16 years, at 6 per cent. per annum?

Ans. 6931 dolls. 57 cts. 5 m.

5. A has B's note for 650 dollars, payable at the end of 20 years, at 6 per cent. per annum, compound interest; what sum will it require to discharge the note, at the expiration of the given time?

Ans. 2084 dolls. 63 cts. 4 m.

6. A father left a legacy of 8000 dollars at compound interest, 6 per cent. per annum, to be equally divided among his three sons, when the youngest, who was 4 years old, should arrive at the age of 21; what will be each one's share? *Ans.* 7180 dolls. 72 cts. each share.

SECTION 4.

Insurance, Commission, and Brokage.

INSURANCE is a premium given for insuring the owners of property against the dangers and losses to which it is liable, or indemnifying for its loss.

The instrument of agreement by which this indemnity is secured is termed *the policy* of insurance.

Commission is a compensation allowed to merchants and others for buying, selling, and transporting goods, wares, &c.

Brokage is an allowance given to brokers for exchanging money, buying and selling stock, &c.

The method of operation in all these is the same as in simple interest.

EXAMPLES.

INSURANCE.

1. What is the premium of insuring 1260 dollars, at 5 per cent?

1260
5

63,00 *Ans.* 63 dolls.

		D.	*c.*
2. 1650 dollars at 15½ per cent.	*Ans.*	255	75
3. 4500 25		1125	00

4. What sum must a policy be taken out for, to cover 900 dollars, when the premium is 10 per cent?

100 policy
10 premium

90 sum covered

As 90 : 100 :: 900 : 1000 dolls. *Ans.*

5. What sum will it require to cover a policy of insurance for 4500 dolls. at 25 per cent? *Ans.* 6000 dolls.

6. What sum will it require to cover a policy of insurance for 560 dollars, at 9 per cent? *Ans.* 615 dolls. 38½ cts.

COMMISSION.

1. What is the commission on 850 dolls. at 5 per cent.

850
5

42,50 *Ans.* 42 dolls. 50 cts.

2. What is the commission on 1260 dollars, at 6 per cent? *Ans.* 75 dolls. 60 cts.

		D.	*c.*
3. [illegible]50 dollars at 4 per cent.	*Ans.*	102	00
3		790	26
¾		48	16½

6. A commission merchant receives 1260 dollars to fill an order, from which he is instructed to deduct his own commission of 5 per cent. how much will remain to satisfy the order?

100
5 per cent.

As 105 : 100 :: 1260 : 1200 dolls. *Ans.*

7. A commission merchant has received 4120 dollars with instructions to vest it in salt at 8 dollars per barrel; deducting from it his commission of 3 per cent. how many barrels of salt can he purchase? *Ans.* 500 barrels.

BROKAGE.

1. What is the brokage on 1000 dollars, at $1\frac{1}{2}$ per cent?

1000
$1\frac{1}{2}$

1000
500

15,00 *Ans.* 15 dolls.

2. What is the brokage on 1625 dollars 50 cents, at $3\frac{1}{2}$ per cent? *Ans.* 54 dolls. 18 cts.

3. 1868 dollars at $2\frac{1}{2}$ per cent. *Ans.* 46 dolls. 70 cts.

4. 560 6 33 60

SECTION 5.

BUYING AND SELLING STOCKS.

Stock is a fund vested by government, or individuals in a corporate capacity, in banks, turnpike roads, bridges, &c. the value of which is subject to rise and fall.

RULE.

Multiply the given sum by the rate per cent. and divide the product by 100

EXAMPLES.

1. What is the amount of 1650 dollars, United States bank stock, at 125 per cent. or 25 per cent. above par?

1650
125

8250
3300
1650

1,00)2062,50 *Ans.* 2062 dolls. 50 cts.

Or thus, 25 is ¼)1650
412 50

2062 50 *Ans.*

	D.				*D.*	*c.*
2.	1500	bank stock at	110	per cent. *Ans.*	1650	00
3.	1686	——	128	- - -	2158	08
4.	25000	——	108	- - -	27000	00
5.	1260	——	90	- - -	1134	00
6.	9254	——	84	- - -	7773	36
7.	1518	——	83¾	- - -	1271	32½

SECTION 6.

REBATE OR DISCOUNT,

Is a reduction made for the payment of money before it becomes due. It is estimated in such a manner, as that the ready payment, if put to interest at the same rate and time, would amount to the first sum. Thus, 6 dollars is the discount on 106 dollars for 12 months, at 6 per cent. leaving 100 dollars the ready payment, which, if put to interest for the same rate and time, would regain the 6 dollars discount.

RULE.

As 100 dollars and the interest for the given time, is to 100 dollars, so is the given sum to its present worth.—Subtract the present worth from the given sum, and the remainder is the discount.

EXAMPLE.

1. What is the discount of 1696 dollars, due 12 months hence, at 6 per cent. per annum?

As 106 : 100 :: 1696 : 1600
1600

96 dolls. *Ans.*

2. What is the present worth of 2464 dollars, due 1 year and 6 months hence, discounting at the rate of 8 per cent. per annum? *Ans.* 2200 dolls.

3. A has B's note for 1857 dollars 50 cents, payable 8 months after date; what is the present worth of said note, discounting at the rate of 5½ per cent. per annum?

Ans. 1791 dolls. 80 cts.

4. What reduction must be made for prompt payment of a note for 650 dollars, due 2 years hence, 7 per cent. per annum being allowed for discount?

Ans. 79 dolls. 83+cts.

5. What is the present worth of 5150 dollars, due in 4½ months, discounting at the rate of 8 per cent. per annum, and allowing 1 per cent. for prompt payment?

Ans. 4950 dolls.

Note. Discount and interest are often supposed to be one and the same thing; and in business, the interest for the time is frequently taken for the discount, and it is presumed neither party sustains any loss. This however is not true, for the interest of 100 dollars for 12 months, at 6 per cent. is 6 dollars, whereas the discount for the same sum, at the same rate and time, is only 5 dollars 66 cents, making a difference of 34 cents for every 100 dollars for 1 year at 6 per cent.—The following examples will show the difference.

EXAMPLES.

1. What is the discount of 1272 dollars, due in 12 months, discounting at 6 per cent. per annum?

As 106 : 100 :: 1272 : 1200
discount 72 dolls.

2. What is the interest of the same sum, for the same time and rate?

1272
6
76,32 interest.

	D.	c.
Interest	76	32
Discount	72	
Difference	4	32

3. What is the difference between the interest and discount on 7280 dollars, for 18 months, at 8 per cent. per annum? *Ans.* 93 dolls. 60 cts. difference.

Note. But when discount is made for present payment, without regard to time, the interest of the sum as calculated for a year, is the discount.

EXAMPLE.

1. How much is the discount of 260 dollars at 5 per cent?

260
5

13,00 *Ans.* 13 dollars.

2. What is the discount on 1650 dollars, at 3 per cent? *Ans.* 49 dolls. 50 cts.

3. What sum will discharge a bond for 2464 dollars, on which a discount of 8 per cent. is given? *Ans.* 2266 dolls. 88 cts.

SECTION 7.

BANK DISCOUNT.

BANK discount is the interest which banks receive for the use of money loaned by them for short periods. And as banks from long established custom, give three days over and above the time limited by the words of the note, called *days of grace;* and as the day of the date, and the day of payment are both calculated, which makes the time 4 days longer than expressed in the note, so interest must be calculated on these days in addition to the regular interest on the given sum, for the specified time.

RULE.

Add 4 to the number of days specified in the note, multiply the given sum by this number, and divide the product by 60. Or,

Multiply the given sum by half the number of days, and di-

Note. When the cents in the given sum are less than 50, the bank loses the interest on them, but when they are more than 50 they charge interest for one dollar.

EXAMPLES.

1. Required the discount of 1500 dollars for 60 days.

		Or,	
1500			1500
64			32=half the days
6000			3000
9000			4500
6,0)9600,0			3,0)4800,0
16,00	*Ans.* 16 dolls.		16,00

2. What is the discount of 250 dollars for 30 days?
Ans. 1 doll. 41⅔ cts.
3. What is the discount of 600 dollars for 90 days?
Ans. 9 dolls. 40 cts.
4. What is the discount of 1260 dollars 40 cents for 60 days?
Ans. 13 dolls. 44 cts.
5. What is the discount of 2649 dolls. 75 cents for 60 days?
Ans. 28 dolls. 26 cts. 4 m.

Form of a note offered for discount.

———— *Pittsburgh, July* —, 1832.

Dollars ————

Sixty days after date, I promise to pay A. B. or order, at the bank of ———, the sum of ——— dollars, without defalcation; value received. J. P.

SECTION 8.

EQUATION OF PAYMENTS.

EQUATION of payments is the finding the *mean time*, for the payment of two or more sums of money payable at different times.

RULE.

Multiply each sum by its own time. Add the products

into one sum and divide this amount by the whole debt; the quotient will be the mean time.

EXAMPLE.

1. A owes B 600 dollars, of which 200 is to be paid at 4 months, 200 at 8 months, and 200 at 12 months; but they agree to make but one payment; when must that payment be made?

$$\begin{array}{lr} 200\times\ 4= & 800 \\ 200\times\ 8= & 1600 \\ 200\times 12= & 2400 \\ \hline 600\,) & 4800\,(\,8 \text{ months. } \textit{Ans.} \\ & 4800 \\ \hline \end{array}$$

2. A merchant has owing to him from his friend, the sum of 3000 dollars, to be paid as follows, *viz.* 500 dollars at 2 months, 1000 dollars at 5 months, and the rest at 8 months; but they agree to make one payment of the whole; what will be the mean time of payment? *Ans.* 6 months.

3. A buys of B 50 acres of land, for which he agrees to pay 1000 dollars at the following times, *viz.* 200 dollars at 5 months, 300 dollars at 8 months, and the rest at 10 months, but an equation of payments is afterwards agreed upon; when must the payment be made?

Ans. 8 months 12 days.

4. C owes D 1400 dollars, to be paid in 3 months, but D being in want of money, C pays him 1000 dollars at the expiration of 2 months; how much longer than 3 months may he in justice defer the payment of the rest?

Ans. 2½ months.

SECTION 9.

FELLOWSHIP.

FELLOWSHIP teaches to find the *profit* or *loss* arising to different partners in trade, in proportion to the capital or stock each has advanced.

Fellowship is either *single* or *compound*.

SINGLE FELLOWSHIP,

Is when the stocks employed are different, but the time alike.

RULE.

Find the amount of the whole stock employed; and then (by proportion) as the whole stock is to the whole gain or loss, so is each partner's stock to his share of the gain or loss.

EXAMPLE.

1. Two merchants join their stock in trade; A puts in 600 dollars, and B puts in 400 dollars, and they gain 250 dollars; what part belongs to each?

A 600	As 1000 : 250 ::	600 to A's share	150	*Ans.*
B 400		400 to B's	100	
1000			250	

2. Three merchants enter into partnership in trade; A advanced 7500 dollars, B 6000, and C 4500, with this they gained 5400 dollars; what was each partner's share?

Ans. A 2250 dolls. B 1800 C 1350

3. A bankrupt is indebted to A 1291 dollars 23 cents, to B 500 dollars 37 cents, to C 709 dollars 40 cents, to D 228 dollars; and his estate is worth but 2046 dollars 75 cents; how much does he pay per cent. and how much is each creditor to receive?

		D.	*c.*
Ans. he pays 75 per cent. and	A receives	968	42½
	B	375	27¾
	C	532	05
	D	171	

4. Three men, A, B, and C, rent a farm containing 585 acres 2 roods and 34 perches, at 600 dollars per year, of which A pays 180 dollars, B 195, and C 225, and they agree that the farm shall be divided in proportion to the rents; how many acres must each man have?

		A.	*R.*	*P.*
Ans.	A's share is	175	2	34½
	B's	190	1	17½
	C's	219	2	22½

5. Three merchants freighted a ship with 2160 barrels of flour, of which 960 barrels belonged to A, 720 barrels to B, and 480 barrels to C; but on account of stormy weather they were obliged to throw 900 barrels overboard; how many barrels did each man lose?

Ans. A lost 400 barrels.
B 300
C 200

6. Three merchants join stock in trade; A put in 1260 dollars, B 840 dollars, and C a certain sum; and they gained 825 dollars, of which C took for his part 275 dollars; required A and B's part of the gain, and how much stock C put in?

Ans. A gained 330 dolls.
B 220
C's stock was 1050

7. Four men traded with a stock of 800 dollars, and they gained in two years time twice as much, and 40 dollars over; A's stock was 140 dollars, B's 260, C's 300; required D's stock, and what each gained?

Ans. D's stock was 100 dolls.
A's gain was 287
B's 533
C's 615
D's 205

8. Three butchers lease a pasture field for 96 dollars, into which they put 300 beef cattle; of these 80 belonged to A, 100 to B, and 120 to C; how much had each to pay?

Ans. A 25 dolls. 60 cts. B 32 dolls. C 38 dolls. 40 cts.

9. A father left an estate of 5000 dollars to his three sons, in such a manner that for every 2 dolls. that A gets, B shall have 3, and C 5; how much did each son receive?

Ans. A gets 1000 dolls. B 1500, C 2500.

10. A, B and C put in money together, A put in 20 dolls B and C together put in 85 dollars; they gained 63 dolls. of which B got 21 dollars; what did A and C gain, and how much did B and C separately put in?

Ans. A gained 12 dolls.
C 30
B put in 35
C 50

COMPOUND FELLOWSHIP.

COMPOUND FELLOWSHIP is when both the stocks and times are different.

RULE.

Multiply each partner's stock by the time it is employed, add all the products into one sum; then say, as the sum of the products is to the whole gain or loss, so is each partner's stock multiplied by the time, to his share of the gain or loss.

EXAMPLE.

1. Three merchants entered into trade; A put in 2500 dollars for 4 months, B 3000 dollars for 6 months, and C 4000 dollars for 8 months, and they gained 1200 dollars; what is each man's share of the gain?

	D.	*M.*	
A	2500	× 4 =	10000
B	3000	× 6 =	18000
C	4000	× 8 =	32000
		Sum	60000

As 60000 : 1200 :: { 10000 : 200 A's share; 18000 : 360 B's; 32000 : 640 C's } *Ans.*

1200 proof.

2. Three merchants enter into partnership for 16 months; A put into stock at first 600 dollars, and at the end of 8 months, 200 dollars more; B put in at first 1200 dollars, but at the end of 10 months, was obliged to take out 600 dollars; C put in at first 1000 dollars, and at the end of 12 months put in 800 more; with this stock they gained 2300 dollars; what was each man's share?

Ans. { A's share is 560 dolls.; B's 780; C's 960 }

3. A and B join stock in trade; A put in 600 dollars on the first of January; B advanced on the first of April a sum

which entitled him to an equal share of the profit at the end of the year; required the sum B put in?

Ans. 800 dollars.

4. D put in stock 1800 dollars; E at the end of 4 months agrees to advance such a sum as at the end of the year will entitle him to an equal share of the profits; what sum must E advance? *Ans.* 2700 dollars.

5. Two gentlemen, A and B, hired a carriage in Pittsburgh to go to Philadelphia, and return, for 160 dollars, with liberty to take in two others by the way. When at Philadelphia they took in C, and afterwards, 100 miles from Pittsburgh, they took in D. Now allowing it to be 300 miles from Pittsburgh to Philadelphia, and also that each man pays in proportion to the distance he rode; it is required to tell how much each must pay?

Ans.
A pays	60 dolls.
B	60
C	30
D	10
	160 proof.

6. Three graziers hired a piece of pasture ground for 145 dolls. 20 cents; A put in 5 oxen for 4½ months, B put in 8 oxen for 5 months, and C put in 9 oxen for 6½ months; how much must each pay?

Ans. A pays 27 dolls. B 48 dolls. and C 70 dolls. 20 cts.

7. A, B, and C have received 665 dollars interest; A put in 4000 dolls. for 12 months, B 3000 for 15 months, and C 5000 for 8 months; how much is each man's part of the interest?

Ans.
A	240 dolls.
B	225
C	200

8. Three merchants lost by some dealings 263 dollars 90 cents; A's stock was 580 dolls. for 6½ months, B's 580 dolls. for 9½ months, and C's 870 dolls. for 8⅔ months; how much is each man's part of this loss?

Ans.
A's loss	59 dolls.	15 cts.
B's	86	45
C's	118	30

SECTION 10.

PROFIT AND LOSS.

By this rule we discover what has been gained or lost on the purchase and sale of goods, and merchandise of every kind.

RULE.

Prepare the question by reduction when necessary, and then work by the *Rule of Three* or *Practice*, as the nature of the question may require.

EXAMPLE.

1. Bought 360 barrels of flour for 6 dollars 25 cents per barrel, and sold it for 7 dollars 50 cents per barrel; what is the profit on the whole?

D.	*c.*
7	50
6	25
1	25 gain per barrel.

B.	*D. c.*		*B.*	*D.*
As 1	: 1 25	::	360	: 450 *Ans.*

2. Bought a piece of cloth for 1 doll. and 20 cents per yard, and sold it again for 1 dollar 50 cents a yard; what is the gain per cent? *Ans.* 25 per cent.

3. Bought a piece of linen containing 42 yards for 21 dollars, and sold it at 66 cents per yard; what is the gain or loss on the whole piece?

Ans. 6 dolls. 72 cents gain.

4. A merchant bought 6 barrels of whiskey containing 32 gallons each, for 96 dollars; while in his possession he lost 12 gallons by leakage, the residue he sold for such a sum as gained him 12 dollars on the whole; how much per gallon did he buy and sell for?

Ans. Bought for 50 cents, and sold for 60 cents per gall.

5. Bought 120 doz. of knives for 20 cents each knife, and sold them again for 17 cents each, what was the loss on the whole? Ans. 43 dolls. 20 cts.

29148

6. A merchant gave 149 dollars for 100 yards of cloth; at how much per yard must he sell it to gain 51 dollars on the whole? *Ans.* 2 dollars.

7. Bought a chest of tea at 1 dollar and 25 cents per pound, but finding it to be of an inferior quality, I am willing to lose 16 per cent. by it; how must I sell it per pound?

Ans. 1 doll. 2½ cents per lb.

8. A merchant bought 20 dozen of wool hats at 90 cents per hat; at what rate must he sell them again to gain 20 per cent. and how much does he gain on the whole?

Ans. he must sell at 1 dollar 8 cents per hat, and gains 43 dollars 20 cents.

9. A trader bought a hogshead of rum of a certain proof, containing 115 gallons, at 1 dollar 10 cents per gallon; how many gallons of water must he put into it to gain 5 dollars, by selling it at 1 dollar per gallon?

Ans. 16½ gallons.

10. A merchant bought 4 hundred weight of coffee for 134 dollars 40 cents, and was afterwards obliged to sell it at 25 cents per pound; what was his loss on the whole, and how much on each pound?

Ans. 5 cents loss on each pound, and 22 dollars 40 cents on the whole.

11. If by selling 360 yards of broadcloth for 1728 dollars, there is gained 20 per cent. profit, what did it cost per yard?

Ans. 4 dollars.

12. A merchant laid out 1000 dollars on cloth, at 4 dollars per yard, and sold it again at 4 dollars 90 cents per yard; what was his whole gain? *Ans.* 225 dolls.

13. A sells a quantity of wheat at 1 dollar per bushel, and gains 20 per cent.; shortly after he sold of the same to the amount of 37 dollars 50 cents, and gained 50 per cent.; how many bushels were there in the last parcel, and at what rate did he sell it per bushel?

Ans. 30 bushels, at 1 doll. 25 cents per bushel.

14. A trader is about purchasing 5000 galls. of whiskey, which he can have at 48 cents per gallon in ready money, or 50 cents with two months credit; which will be the most profitable, either to buy it on credit, or by borrowing the money at 8 per cent. per annum, to pay the cash price?

Ans. he will gain 68 dollars by paying the cash.

15. A butcher bought 12 head of beef cattle of equal

weight, for 240 dollars, which he sells again for 4 cents per pound; what ought each one to weigh, that the butcher may have the hides and tallow as clear gain?

Ans. 4*cwt.* 1*qr.* 24*lb.*

SECTION 11.

BARTER.

BARTER is the exchanging of one commodity for another at the rates agreed upon by their owners.

RULE.

Proceed by the rules of reduction and proportion, as the nature of the question may require.

EXAMPLE.

1. How many yards of linen at 50 cents per yard must be given for 6¼ yards of broadcloth, at 4 dollars 50 cents per yard?

4,50 dollars
6¼

2700
112½

28,12½

c.	*yd.*	*D. c.*	*yds.*
As 50	: 1	:: 28,12½	: 56¼ *Ans.*

2. A has 320 bushels of salt at 1 dollar 20 cents per bushel, for which B agrees to pay him 160 dollars in cash and the rest in coffee at 20 cents per pound; how much coffee must A receive? *Ans.* 1120 lb.

3. How much rye at 70 cents per bushel must be given for 28 bushels of wheat, at 1 dollar 25 cents per bushel?

Ans. 50 bushels.

4. A barters 319 lb. of coffee at 23½ cents per pound, with B for 250 yards of muslin; what does the muslin cost A per yard? *Ans.* 30 cents nearly.

5. C has flour at 5 dollars per barrel, which he barters

to D at a profit of 20 per cent. for tea which cost 1 dollar 25 cents per pound; at what rate must D sell the tea to make the barter equal? *Ans.* 1 doll. 50 cts. per lb.

6. A has cloth which cost him 2 dollars 50 cents per yard, but in trade he must have 2 dollars 80 cents; B has wheat at 1 dollar 20 cents per bushel; at how much per bushel should he sell to A, to make the barter equal?

Ans. 1 doll. $34\frac{2}{5}$ cents.

7. P has 240 bushels of rye which cost him 90 cents per bushel; this he barters with Q at 95 cents per bushel for wheat which stands Q 99 cents per bushel; how many bushels of wheat is he to receive in barter, and at what price, that their gains may be equal?

Ans. $218\frac{2}{11}$ bushels, at 1 doll. $4\frac{1}{2}$ cts. per bushel.

8. A gives B in barter 26*lb.* 4*oz.* of cinnamon, at 1 dollar 28 cents per pound, for rice at 6 cents per pound; how much rice must A receive? *Ans.* 5 cwt.

9. C and D barter; C has muslin that cost him 22 cents per yard, and he puts it at 25 cents; D's cost him 28 cents per yard; at what price must he put it to gain 10 per cent. more than C? *Ans.* $34\frac{34}{35}$ cents per yard.

10. A buys 250 barrels of flour from B, at 6 dollars 25 cents per barrel, in payment B takes 4 cwt. of coffee at 30 cents per pound, 64 pounds of tea at 1 dollar 75 cents per lb. 25 yards of broadcloth at 6 dollars per yard, 206 dollars 10 cents in cash, and the balance in salt, at 8 dollars per barrel; how many barrels of salt must B receive?

Ans. 120 barrels.

SECTION 12.

EXCHANGE.

EXCHANGE is the reducing the money, coin, &c. of one state or country to its equivalent in another.

Par is equality in value; but the course of exchange is often above or below *par*.

Agio is a term sometimes used, to express the difference between bank and current money.

Case 1.

To change the currency of one state into that of another.

RULE.

Work by the Rule of Three; or by the theorems in the following table:—

TABLE, exhibiting the Value of a Dollar in each of the United States; and practical Theorems for Exchanging the Currency of either into that of any other.

To exchange from ... to	*N. England States, Virginia, Kentucky, Ohio, Tennesse, Indiana & Miss.*	*Pennsylvania, New Jersey, Delaware & Maryland.*	*New York and North Carolina.*	*South Carolina and Georgia.*
*New England States, Virginia, &c.	Dollar 6*s.*	Add one 4th.	Add one 3d.	Subtract $\frac{1}{9}$ twice.
Pennsylvania, New Jersey, Delaware & Maryland,	Subtract one 5th.	Dollar 7*s.* 6*d.*	Add one 15th.	$\times 3\frac{1}{3}$ & $\div 5$
New York & North Carolina,	Subtract one 4th.	Subtract one 16th.	Dollar 8*s.*	To $\frac{1}{9}$ add $\frac{1}{8}$ of the $\frac{1}{9}$
South Carolina and Georgia,	Add two 7ths.	Add $\frac{1}{7}$ that $\frac{1}{7}$ and $\frac{1}{8}$ that $\frac{1}{7}$.	$\times 2$ and Subtract $\frac{1}{9}$ Product	Dollar 4*s.* 8*d.*

* The New England States are, New Hampshire, Massachusetts, Rhode Island, Connecticut, Vermont, and Maine.

Note.—In some parts of Ohio and Tennesee, the dollar passes for 7*s.* 6*d.*, and in the new states, viz: Louisiana, Missisippi, Illinois, and Alabama, accounts are generally kept in federal money.

The value of a dollar in any state is found, either opposite to that state, or under it in the table.

EXAMPLE.

1. What is the value of 480*l.* Pennsylvania currency in North Carolina?

	s.	*d.*		*s.*		£.		£.	
As	7	6	:	8	::	480	:	512	*Ans.*

	£.
Or,	480
Add $\frac{1}{7}$ =	32
	512 *Ans.*

2. What is the value of 256*l.* New York currency in Pennsylvania? *Ans.* 240*l.*

3. How much South Carolina currency is equal to 1500*l.* of New Jersey? *Ans.* 933*l.* 6*s.* 8*d.*

4. What sum New York currency is equal to 180*l.* in Massachusetts? *Ans.* 240*l.*

5. How much Virginia currency will purchase a bill for 280*l.* South Carolina? *Ans.* 360*l.*

6. A bill of exchange being remitted from Rhode Island to South Carolina for 304*l.*, what is its value in the currency of the latter? *Ans.* 236*l.* 8*s.* $10\frac{1}{2}$*d.*+

Case 2.

To change the currency of the different states to Federal money.

RULE.

Divide the given sum, reduced to shillings, six-pences, or pence in a dollar, as it passes in each state.

EXAMPLES.

1. Change 127*l.* 12*s.* New England money to dollars and cents.

127*l.* 12*s.*=2552 shillings.

The dollar, New England, is 6*s.*) 2552

425,$33\frac{1}{3}$

Ans. 425 dolls. $33\frac{1}{3}$ cts.

2. Change 37*l.* 10*s.* Pennsylvania currency, to dollars.

37*l.* 10*s.*=1500 six-pences.

7*s.* 6*d.* or 15 six-pences make a dollar Pennsylvania currency; hence 1500÷15=100 dolls. *Ans.*

Or, 37*l.* 10*s.*=9000 pence=100,00 cents. *Ans.*

3. Change 225*l.* 12*s.* New York currency to Federal money.

225*l.* 12*s.*=4512 shillings ÷ 8 the dollar New York currency=564 dollars. *Ans.*

4. A bill of exchange for 468*l.* 9*s.* 6*d.* Virginia currency, is remitted to Philadelphia; what is its value in Federal money? *Ans.* 1563 dollars 25 cts.

5. A merchant deposited in the United States branch bank at Pittsburgh, the sum of 750*l.* 10*s.* Pennsylvania currency, for what sum may he draw for in Federal money? *Ans.* 2001 dollars 33⅓ cents.

Note.—Federal money being now generally introduced into mercantile business throughout the United States, has nearly superseded the use of the above rules of exchange between the different States.

Case 3.

FOREIGN EXCHANGE.

Accounts are kept in England, Ireland, and the West India Islands, in pounds, shillings, pence, and farthings though their intrinsic value in these places is different.

A TABLE

Of different Moneys, as they are denominated and valued in different countries.

GREAT BRITAIN, IRELAND, AND THE WEST INDIES.

4 farthings	=1 penny
12 pence	1 shilling
20 shillings	1 pound

FRANCE.

12	Deniers	= 1 Sol
20	Sols	1 Livre
3	Livres	1 Crown

SPAIN.

4	Marvadies Vellon, or 2½ Marvadies of Plate	= 1 Quarta
8½	Quartas, or 34 Marvadies Vellon	1 Rial Vellon
16	Quartas, or 34 Marvadies of Plate	1 Rial of Plate
8	Rials of Plate	1 Piaster, Pezo, or Dollar
5	Piasters	1 Spanish Pistole
2	Spanish Pistoles	1 Doubloon

ITALY.

12	Deniers	= 1 Sol
20	Sols	1 Livre
5	Livres	1 Piece of Eight at Genoa
6	Livres	1 Ditto at Leghorn
6	Solidi	1 Gross
24	Grosses	1 Ducat

PORTUGAL.

400	Reas	= 1 Crusadoe
1000	Reas	1 Millrea

HOLLAND.

8	Penning	= 1 Groat
2	Groats	1 Stiver=2*d.*
6	Stivers	1 Shilling
20	Stivers	1 Florin, or Guilder
2½	Florins	1 Rix Dollar
6	Florins	1 £. Flemish
5	Guilders	1 Ducat

DENMARK.

16	Shillings	= 1 Mark
6	Marks	1 Rix Dollar
32	Rustics	1 Copper Dollar
6	Copper Dollars	1 Rix Dollar

RUSSIA.

18	Pennins	= 1 Gros
30	Gros	1 Florin
3	Florins	1 Rix Dollar
2	Rix Dollars	1 Gold Ducat

RULE.

In exchanging of foreign moneys, work by the Rule of Three, or by Practice; and for exchanging foreign moneys to Federal, work by the table in page 40.

EXAMPLES.

1. Philadelphia is indebted to London 1749*l.* currency, what sum sterling must be remitted, when the exchange is 65 per cent.?

	£.		£.		£.		£.	
As	165	:	100	::	1749	:	1060 sterling.	*Ans.*

2. London is indebted to Philadelphia 1060*l.* sterling; what sum Pennsylvania currency must be remitted, the exchange being 65 per cent. as above?

	£.		£.		£.		£.	
As	100	:	165	::	1060	:	1749	*Ans.*

Or,	50	$\frac{1}{2}$	1060
	10	$\frac{1}{5}$	530
	5	$\frac{1}{2}$	106
			53

1749*l.* *Ans.*

3. *Baltimore, Oct.* 1, 1817.

Exchange for 1260*l.* 10*s.* sterling.

Thirty days after sight of this my first of exchange, second and third of like tenor and date not being paid, pay to A. B. or order, twelve hundred and sixty pounds ten shillings sterling, value received, and place the same to account, as per advice from

P— S—n.

W. L. merchant, London.

What is the value of this bill in Federal money?

1260*l.* 10*s.*=1260,5. × by 444 cents=5596 dolls. 62 cents. *Ans.*

4. *London, January* 1, 1818.

Exchange for 5596 dolls. 62 cts. Federal money.

Thirty days after sight of this my second of exchange, first and third of the same tenor and date not paid, pay to J. B. or order, five thousand five hundred and ninety-six dollars sixty-two cents, value received, and place the same to account, as per advice from

S. S.

Mr. T. L. merchant, Baltimore.

How much sterling is the above bill, 4,44 cents to the pound?

444)5596,62(1260
444
1156
888
2686
2664
222
20
4440(10 *Ans.* 1260*l.* 10*s.*
4440

5. A merchant of Philadelphia receives from his correspondent in Dublin, a bill of exchange for 540*l.* 15*s.* Irish currency; what is its value in Federal money?

Ans. 2217 dolls. 7½ cts.

6. A merchant in Philadelphia draws on his correspondent in Dublin for the balance of an account amounting to 2217 dolls. 7½ cents; what sum Irish currency must be remitted to satisfy the draft? *Ans.* 540*l.* 15*s.*

Note.—In these last examples the course of exchange is considered as being at par: when the exchange is above or below par, the per cent. must be added or subtracted, as the case requires.

7. In a settlement between A of London and B of Philadelphia, B is indebted to A in the sum of 320*l.* sterling, what sum must be remitted by B to A to settle the balance, the exchange being 12½ per cent. from the United States to Great Britain? *Ans.* 1598 dolls. 40 cts.

8. C of New York remits 3259 dollars to his correspondent in Dublin, to be placed to his account; for what sum Irish currency, must he receive credit, the course of exchange being 8 per cent. in favor of Ireland?

Ans. 736*l.* nearly.

Note. The par of exchange between the United States of America and most other trading countries, may be found by the table in page 40.

SECTION 13.

ALLIGATION.

ALLIGATION is a rule for finding the prices, and quantity of simples in any mixture compounded of those things.

Case 1.

To find the mean price of any part of the composition, when the several quantities and prices are given.

RULE.

As the sum of the whole quantity, is to its total value, so is any part of the composition, to its value.

EXAMPLE.

1. A merchant mixed 2 gallons of wine at 2 dollars per gallon, 2 at 2 dollars 50 cents, and 2 at 3 dollars; what is one gallon of this mixture worth?

gal.	
2 at	2,00=400
2 at	2,50=500
2 at	3,00=600
6	1500

G.	*D. c.*		*G.*		*D. c.*	
As 6	: 15,00	::	1	:	2,50	*Ans.*

2. A grocer mixed 20 lb. of sugar at 10 cents per lb. 30 lb. at 15 cents, and 40 lb. at 25 cents; what is one pound of this mixture worth? *Ans.* 18⅓ cts.

3. A trader mixes 10 bushels of salt at 150 cents, 20

160 cts. and 30 at 170 cts. per bushel; at what rate can he afford to sell one bushel of this mixture? *Ans.* 163⅓ cts.

4. If 4 ounces of silver at 75 cents per ounce, be melted with 8 ounces at 60 cents per ounce, what is the value of one ounce of this mixture? *Ans.* 65 cents.

Case 2.

To find what quantity of several simples must be taken at their respective rates, to make a mixture worth a given price.

RULE.

Place the rates of the simples under each other, and link each rate which is less than the mean rate, with one or more that is greater. The difference between each rate and the mean price set opposite to the respective rates with which it is linked, will be the several quantities required.

Note. 1. If all the given prices be greater or less than the mean rate, they must be linked to a cipher.

2. Different modes of linking will produce different answers.

EXAMPLES.

1. How many pounds of tea at 150, 160, and 200 cents per pound, must be mixed together, that 1 pound may be sold for 180 cents?

Mean rate 180 { 150, 160, 200 } 30+20=50 { 20 at 150 cents, 20 at 160, } *Ans.*

2. How many gallons of wine at 3, 5, and 6 dollars per gallon, must be mixed together, that one gallon may be worth 4 dollars?

Ans. 3 gallons at 3 dolls. 1 gallon at 5 dolls. and 1 gallon at 6 dollars.

3. How many bushels of rye at 40 cents per bushel, and corn at 30 cents, must be mixed with oats, at 20 cents, to make a mixture worth 25 cents per bushel?

1. *Ans.* 25 { 20, 30, 40 } 15+5 5 2. *Ans.* { 6 bushels of rye, 6 do. corn, 24 do. oats }

4. A grocer has four several sorts of tea, viz. one kind at 120 cents, another at 110 cents, another at 90 cents

and another at 80 cents per pound, how much of each sort must be taken to make a mixture worth 1 dollar per pound?

1. *Ans.* { 2 at 120 cents. / 1 110 / 1 90 / 2 80 }

2. *Ans.* { 3 at 120 cents. / 2 110 / 2 90 / 3 80 }

3. *Ans.* { 1 at 120 cents. / 2 110 / 2 90 / 1 80 }

4. *Ans.* { 1 at 120 cents. / 3 110 / 3 90 / 1 80 }

5. *Ans.* { 3 at 120 cents. / 1 110 / 3 90 / 2 80 }

6. *Ans.* { 2 at 120 cents. / 3 110 / 1 90 / 3 80 }

Note. From this last example it is manifest that a great many different answers may result to the same question, according to the various modes of linking the numbers together.

Case 3.

When the rate of all the simples, the quantity of one of them, and the compound rate of the whole mixture are given, to find the several quantities of the rest.

RULE.

Arrange the mean rate, and the several prices, linked together as in case 2, and take their difference.

Then, as the difference of the same name with the quantity given,

Is to the rest of the differences respectively:

So is the quantity given,

To the several quantities required.

EXAMPLE.

1. A grocer would mix 40 pounds of sugar at 22 cents per pound, with some at 20, 14, and 12 cents per pound; how much of each sort must he take to mix with the 40 pounds, that he may sell the mixture at 18 cents per pound?

18 { 12 —4 lb. / 14 —2 / 20 —4 / 22 —6 against the price of the given quantity.

As 6 : 40 :: 4 : 26,66 lb. at 12 cents.
6 : 40 :: 2 : 13,33 do. 14 } Ans.
and 26,66 do. 20

2. How much wheat at 48 cents, rye at 66 cents, and barley at 30 cents per bushel, must be mixed with 24 bushels of oats at 18 cents per bushel, that the whole may rate at 22 cents per bushel? *Ans.* 2 bushels of each.

3. How much gold at 16, 20, and 24 carats fine, and how much alloy must be mixed with 10 ounces of 18 carats fine, that the composition may be 22 carats fine?

Ans. 10 oz. of 16 carats fine, 10 of 20, 170 of 24, and 10 of alloy.

Case 4.

When the price of all the simples, the quantity to be mixed, and the mean price are given, to find the quantity of each simple.

RULE.

Find their differences by linking as before:
Then, as the sum of the differences,
Is to the quantity to be compounded;
So is the difference opposite to each price,
To the quantity required.

EXAMPLE.

1. How much sugar at 10, 12, and 15 cents per pound, will be required to make a mixture of 40 pounds, worth 13 cents per pound?

13 { 10 — 2
12 — 2
15 — 3+1=4

8 sum of the different simples.

As 8 : 40 :: 2 : 10 lb. at 10 cts.
8 : 40 :: 4 : 20 do. 15
and 10 do. 12
} *Ans.*

2. How much gold of 15, of 17, of 18, and of 22 carats fine, must be mixed together to form a mixture of 40 ounces of 20 carats fine?

Ans. 5 oz. of 15, of 17, and of 18, and 25 oz. of 22.

3. How many gallons of water must be mixed with wine at 6 dollars per gallon, to fill a vessel of 70 gallons, so that it may be sold without loss at 5 dollars per gallon?

Ans. $11\frac{2}{3}$ gallons of water.

PART VI.

VULGAR FRACTIONS.

A VULGAR FRACTION is any supposed part or parts of an *unit*, and is represented by two numbers placed one above the other, with a separating line between them; thus, $\frac{1}{5}$ one-fifth, $\frac{4}{9}$ four-ninths.

The number above the line is called the *numerator*, and that below the line the *denominator*. Thus,

4 numerator 6
— — &c.
9 denomina. 10

The denominator shows how many parts the unit or integer is divided into, and the numerator shows how many of those parts are contained in the fraction.

Vulgar fractions are either proper, improper, compound, or mixed.

A proper fraction is when the numerator is less than the denominator, as $\frac{1}{2}$, $\frac{2}{3}$, $\frac{4}{5}$, $\frac{12}{17}$, &c.

An improper fraction is when the numerator is either equal to, or greater than the denominator, as $\frac{6}{6}$, $\frac{9}{5}$, $\frac{20}{7}$, &c.

A compound fraction is a fraction of a fraction, as $\frac{2}{3}$ of $\frac{3}{4}$, $\frac{5}{6}$ of $\frac{9}{16}$ of $\frac{12}{15}$, &c.

A mixed fraction is a whole number and fraction united, as $8\frac{3}{5}$, $4\frac{3}{4}$, $120\frac{4}{5}$, &c.

SECTION 1.

Reduction of Vulgar Fractions.

Case 1.

To Reduce a Vulgar Fraction to its lowest terms.

RULE.

Divide the greater term by the less, and that divisor by the remainder, till nothing be left, the last divisor is the

common measure, by which divide both parts of the fr tion: the quotient will be the answer. Or,

Take aliquot parts of both terms continually, till the fr tion is in its lowest terms.

Note. 1. If the common measure when found is 1, the fractio already in its lowest terms.

2. Ciphers to the right of both the terms may be cut off t $\frac{8}{70}=\frac{4}{7}$.

EXAMPLES.

1. Reduce $\frac{36}{48}$ to its lowest terms.

36)48(1
36
—

Common measure 12)36(3
36
—

12)36(3
36
—

12)48(4
48
—

$\frac{3}{4}$ *Ans.*

2 div. 6 div.
Or, $\frac{36}{48}=\frac{18}{24}=\frac{3}{4}$ *Ans.*

2. Reduce $\frac{144}{12}$ to its lowest terms. *Ans.*
3. Reduce $\frac{75}{125}$ to its lowest terms.
4. Reduce $\frac{4300}{10800}$ to its lowest terms.
5. Reduce $\frac{91}{117}$ to its lowest terms.
6. Reduce $\frac{9376}{8684}$ to its lowest terms.

Case 2.

To reduce a mixed number to an improper fraction.

RULE.

Multiply the whole number, by the denominator of fraction, and add the numerator to the product, for a numerator, under which place the given denominator.

EXAMPLE.

1. Reduce $8\frac{3}{4}$ to an improper fraction.

8
4
—
$32+3=\frac{35}{4}$ *Ans.*

2. Reduce $12\frac{15}{17}$ to an improper fraction. *Ans.* $\frac{219}{17}$
3. Reduce $183\frac{5}{21}$ to an improper fraction. *Ans.* $\frac{3848}{21}$
4. Reduce $514\frac{5}{16}$ to an improper fraction. *Ans.* $\frac{8229}{16}$
5. Reduce $68425\frac{3}{4}$ to an improper fraction. *Ans.* $\frac{273703}{4}$

Case 3.

To reduce an improper fraction to a whole or mixed number.

RULE.

Divide the numerator by the denominator; the quotient will be the answer required.

Note. This case and case 2, prove each other.

EXAMPLE.

1. Reduce $\frac{35}{4}$ to its proper terms.

$$4\,)\,35\,(\,8\tfrac{3}{4} \quad \textit{Ans.}$$
$$32$$
$$\overline{3}$$

2. Reduce $\frac{3848}{21}$ to its proper terms. *Ans.* $183\frac{5}{21}$
3. Reduce $\frac{2465}{7}$ to its proper terms. *Ans.* $352\frac{1}{7}$
4. Reduce $\frac{261}{17}$ to its proper terms. *Ans.* $56\frac{9}{17}$
5. Reduce $\frac{8229}{16}$ to its proper terms. *Ans.* $514\frac{5}{16}$

Case 4.

To reduce several fractions to others that shall have one common denominator, and still retain the same value.

RULE.

Reduce the given fractions to their lowest terms, then multiply each numerator into all the denominators, but its own, for a new numerator; and all the denominators into each other for a common denominator.

EXAMPLE.

1. Reduce $\frac{1}{2}$, $\frac{2}{3}$, and $\frac{3}{4}$, to a common denominator.

$1\times3\times4=12$
$2\times2\times4=16$ } numerators.
$3\times2\times3=18$

$2\times3\times4=24$ common denominator.

Ans. $\frac{12}{24}$, $\frac{16}{24}$, $\frac{18}{24}$.

2. Reduce $\frac{3}{4}$, $\frac{4}{5}$, and $\frac{5}{6}$, to a common denominator.

Ans. $\frac{90}{120}$, $\frac{96}{120}$, $\frac{100}{120}$.

3. Reduce $\frac{1}{3}$, $\frac{3}{5}$, $\frac{4}{15}$, and $\frac{5}{6}$, to a common denominator.

Ans. $\frac{075}{225}$, $\frac{135}{225}$, $\frac{060}{225}$, $\frac{187}{225}$.

Case 5.

To reduce several fractions to others, retaining the same value, and that shall have the least common denominator.

RULE.

Divide the given denominators by any number that will divide two or more of them without a remainder; set the quotients and undivided numbers underneath; divide these numbers in the same manner, and continue the operation, till no two numbers are left capable of being lessened; the product of these remaining numbers, together with the divisor or divisors, will give the least common denominator.

Divide the common denominator, so found, by each particular denominator, and multiply the quotient by its own numerator for a new numerator, under which place the common denominator.

EXAMPLE.

1. Reduce $\frac{1}{2}$, $\frac{2}{3}$, $\frac{5}{6}$, and $\frac{7}{8}$, to the least common denominator.

3)2 3 6 8

2)2 1 2 8

1 1 1 4×2×3=24 common denominator.

Divisors { 2)24 ; 12×1=12 ; 3 8×2=16 ; 6 4×5=20 ; 8 3×7=21

Then, $\frac{12}{24}$, $\frac{16}{24}$, $\frac{20}{24}$, $\frac{21}{24}$ *Ans.*

2. Reduce $\frac{2}{3}$, $\frac{3}{4}$, $\frac{4}{12}$, and $\frac{5}{20}$, to the least common denominator.

Ans. $\frac{80}{240}$, $\frac{45}{240}$, $\frac{40}{240}$, $\frac{24}{240}$.

3. Reduce $\frac{1}{2}$, $\frac{3}{5}$, $\frac{4}{15}$, and $\frac{5}{9}$, to the least common denominator. *Ans.* $\frac{15}{45}$, $\frac{27}{45}$, $\frac{12}{45}$, $\frac{25}{45}$

Case 6.

To reduce a compound fraction to a single one.

RULE.

Multiply all the numerators together for a new numerator, and all the denominators for a new denominator.

Note. Such figures as are alike in the numerators and denominators may be cancelled.

EXAMPLE.

1. Reduce $\frac{2}{3}$ of $\frac{3}{4}$ of $\frac{4}{5}$ to a single fraction.

$$\frac{2\times3\times4=24}{3\times4\times5=60} = \frac{2}{5} \text{ Ans.}$$

Or cancelled.

$$\frac{\overset{\cdot\cdot}{2}\ \overset{\cdot}{3}\ \overset{\cdot}{4}}{\underset{\cdot}{3}\ \underset{\cdot\cdot}{4}\ 5} = \frac{2}{5} \text{ Ans.}$$

2. Reduce $\frac{7}{8}$ of $\frac{4}{6}$ of $\frac{9}{10}$ to a single fraction. *Ans.* $\frac{21}{40}$

3. Reduce $\frac{5}{8}$ of $\frac{4}{6}$ of $\frac{3}{4}$ to a single fraction. *Ans.* $\frac{5}{24}$

4. Reduce $\frac{5}{9}$ of $\frac{4}{7}$ of $1\frac{1}{2}$ to a single fraction. *Ans.* $\frac{5}{21}$

Case 7.

To reduce a fraction of one denomination to the fraction of another, but greater, retaining the same value.

RULE.

Make it a compound fraction, by comparing it with all the denominations between it and that to which it is to be reduced; reduce this fraction to a single one.

EXAMPLE.

1. Reduce $\frac{5}{6}$ of a penny to the fraction of a pound.

$$\frac{5}{6} \times \frac{1}{12} \times \frac{1}{20} = \frac{5}{1440} = \frac{1}{288} \text{ Ans.}$$

2. Reduce $\frac{1}{2}$ of a pennyweight to the fraction of a pound troy *Ans.* $\frac{1}{3600}$

3. Reduce $\frac{2}{3}$ of a pint of wine to the fraction of a hogshead. Ans. $\frac{1}{378}$

4. Reduce $\frac{10}{11}$ of a minute to the fraction of a day.

Ans. $\frac{1}{1584}$

Case 8.

To reduce the fraction of one denomination to the fraction of another, but less, retaining the same value.

RULE.

Multiply the given numerator, by the parts of the denominator, between it and that to which it is to be reduced, for a new numerator, and place it over the given denominator, which reduce to its lowest terms.

EXAMPLE.

1. Reduce $\frac{5}{1440}$ of a pound to the fraction of a penny.

$$\frac{5 \times 20 \times 12}{1440 \times 1 \times 1} = \frac{1200}{1440} = \frac{5}{6} \text{ } \textit{Ans.}$$

2. Reduce $\frac{1}{300}$ of a pound troy to the fraction of a pennyweight. *Ans.* $\frac{4}{1}$

3. Reduce $\frac{1}{126}$ of a hogshead to the fraction of a pint.

Ans. $\frac{8}{13}$

4. Reduce $\frac{1}{1584}$ of a day to the fraction of a minute.

Ans. $\frac{10}{11}$

Case 9.

To find the value of a fraction in the known parts of an integer.

RULE.

Multiply the numerator by the known parts of the integer, and divide by the denominator.

EXAMPLE.

1. What is the value of $\frac{2}{3}$ of a pound sterling?

20 shillings = 1 pound.

2

3) 40

13 4 *Ans.* 13*s.* 4*d.*

ace $\frac{3}{8}$ of a pound troy to its proper quantity.

Ans. 7*oz.* 4*dwt.*

3. Reduce $\frac{4}{5}$ of a mile to its proper quantity.
Ans. 6*fur.* 16*p.*

4. Reduce $\frac{3}{10}$ of a day to its proper time.
Ans. 7*h.* 12*min.*

5. What is the value of $\frac{4}{5}$ of a dollar. *Ans.* 80 cts.

Case 10.

To reduce any given quantity, to the fraction of a greater denomination of the same kind.

RULE.

Reduce the given quantity to the lowest denomination mentioned for a new numerator, under which set the integral part (reduced to the same name) for a denominator.

EXAMPLES.

1. Reduce 6*s.* 8*d.* to the fraction of a pound.

s.	*d.*		*s.*
6	8		20
12			12
80	1		240

$\frac{80}{240} = \frac{1}{3}$ *Ans.*

2. Reduce 25 cents to the fraction of a dollar.

$$\frac{25}{100} = \frac{1}{4}$$

3. Reduce 31 gallons 2 quarts to the fraction of a hogshead. *Ans.* $\frac{1}{2}$.

4. Reduce 6 hundred weight 2 quarters 18$\frac{3}{4}$ pounds to the fraction of a ton. *Ans.* $\frac{1}{2}$.

Case 11.

To reduce a vulgar fraction to a decimal of the same value.

RULE.

Add ciphers to the right-hand of the numerator, and divide by the denominator.

EXAMPLE.

1. Reduce $\frac{3}{4}$ to a decimal fraction of the same value.

4)300

,75 *Ans.*

2. Reduce $\frac{17}{20}$ to a decimal fraction. *Ans.* ,85

SECTION 2.

ADDITION OF VULGAR FRACTIONS.

Case 1.

To add fractions that have the same common denominator.

RULE.

Add all the numerators together, and divide the amount by the common denominator.

EXAMPLE.

1. Add $\frac{1}{12}$, $\frac{5}{12}$, $\frac{7}{12}$, $\frac{9}{12}$ and $\frac{11}{12}$ together.

numerators.

1
5
7
9
11

common denominator 12)33(2$\frac{9}{12}$ *Ans.*

24

9 3

$\frac{9}{12} = \frac{3}{4}$

2. Add $\frac{4}{11}$, $\frac{8}{11}$, $\frac{12}{11}$, $\frac{16}{11}$, and $\frac{18}{11}$ together. *Ans.* 2$\frac{7}{11}$

3. Add $\frac{14}{16}$, $\frac{11}{16}$, $\frac{15}{16}$, and $\frac{15}{16}$ together. 2$\frac{7}{8}$.

Case 2.

To add fractions having different denominators.

RULE.

Reduce the given fractions to a common denominator, by case 5, and proceed as in the foregoing case.

EXAMPLE.

1. Add $\frac{1}{2}$, $\frac{3}{4}$, $\frac{5}{8}$ and $\frac{9}{12}$ together.

$$\begin{array}{cccc} 1 & 3 & 5 & 9 \\ - & - & - & - \\ 2 & 4 & 8 & 12 \end{array} = \begin{array}{cccc} 12 & 18 & 15 & 18 \\ - & - & - & - \\ 24 & 24 & 24 & 24 \end{array}$$

$$\begin{array}{r} 12 \\ 18 \\ 15 \\ 18 \\ \hline 24)\,63\,(2\frac{15}{24} \\ \underline{48} \\ 15 \end{array}$$

2. Add $\frac{1}{2}$, $\frac{1}{4}$, $\frac{1}{5}$ and $\frac{1}{8}$ together. *Ans.* $1\frac{3}{40}$

3. Add $\frac{1}{2}$, $\frac{1}{3}$, $\frac{6}{7}$, $\frac{2}{3}$ and $\frac{8}{13}$ together. $3\frac{67}{1673}$

Case 3.

To add mixed numbers.

RULE.

Add the fractions as in the foregoing cases, and the integers as in addition of whole numbers.

EXAMPLES.

1. Add $13\frac{1}{13}$, $9\frac{4}{13}$ and $3\frac{7}{13}$ together.

$$\begin{array}{r} 13\frac{1}{13} \\ 9\frac{4}{13} \\ 3\frac{7}{13} \\ \hline 25\frac{12}{13}=\frac{1}{1}\ \textit{Ans.} \end{array}$$

2. Add $5\frac{2}{3}$, $6\frac{7}{11}$ and $4\frac{1}{2}$ together.

$5\frac{2}{3}=5\frac{44}{66}$ common denominator

$6\frac{7}{11}=6\frac{42}{66}$

$4\frac{1}{2}=4\frac{33}{66}$

$15\frac{119}{66}$ $\qquad$ $\frac{119}{66}=1\frac{53}{66}$

$17\frac{53}{66}$ *Ans.*

3. Add $1\frac{3}{5}$, $\frac{4}{5}$ of $\frac{1}{3}$, and $9\frac{3}{10}$ together. *Ans.* $11\frac{1}{6}\frac{1}{0}$
4. Add $1\frac{9}{10}$, $6\frac{7}{8}$, $\frac{2}{3}$ of $\frac{1}{2}$, and $7\frac{1}{2}$ together. $16\frac{7}{12}\frac{3}{0}$

Case 4.

To add fractions of several denominations.

RULE.

Reduce them to their proper quantities by case 10 in reduction, and add them as before.

EXAMPLE.

1. Add $\frac{7}{9}$ of a £. and $\frac{3}{10}$ of a shilling.

	s.	d.	
$\frac{7}{9}$ of a £. =	15	$6\frac{2}{3}$	$=\frac{10}{15}$ common denom.
$\frac{3}{10}$ of a s. =	0	$3\frac{3}{5}$	$=\frac{9}{15}$
	15	9	$\frac{19}{15}=1\frac{4}{15}$
		$1\frac{4}{15}$	

Shillings 15 $10\frac{4}{15}$ *Ans.*

2. Add $\frac{1}{2}$ of a yard to $\frac{2}{3}$ of a foot. *Ans.* 2 feet 2 inches.
3. Add $\frac{1}{3}$ of a day to $\frac{1}{2}$ of an hour.
Ans. 8 hours 30 minutes.
4. Add $\frac{1}{3}$ of a week, $\frac{1}{4}$ of a day, and $\frac{1}{2}$ of an hour together. *Ans.* 2 days 14 hours 30 minutes.
5. Add $\frac{7}{8}$ of a mile, $\frac{2}{3}$ of a yard, and $\frac{2}{3}$ of a foot together
Ans. 1540 yards 2 feet 9 inches.

SECTION 3.

Subtraction of Vulgar Fractions.

RULE.

PREPARE the fractions as in addition, and subtract the lower numerator from the upper, and place the difference over the common denominator.

Note. 1. When the lower numerator is greater than the upper, subtract it from the common denominator, adding the upper numerator to the difference, and carry 1 to the units place of the integer.

2. When the fractions are of different integers, find their values separately, and subtract as in compound subtraction of whole numbers.

EXAMPLES.

From $\frac{7}{8}$	From $\frac{6}{7}$	From $\frac{2}{3}=\frac{8}{12}$
Take $\frac{3}{8}$	Take $\frac{3}{7}$	Take $\frac{1}{4}=\frac{3}{12}$
Rem. $\frac{4}{8}=\frac{1}{2}$ *Ans.*	Rem. $\frac{3}{7}$ *Ans.*	Rem. $\frac{5}{12}$ *Ans.*

From $\frac{11}{12}$	From $\frac{5}{6}$	From $\frac{209}{216}$	From $\frac{15}{16}$
Take [illegible]	Take $\frac{4}{5}$	Take $\frac{7}{144}$	Take $\frac{11}{12}$
Rem. [illegible]	Rem. $\frac{1}{30}$	Rem. $\frac{397}{432}$	Rem. $\frac{1}{48}$

From [illegible]	From $13\frac{1}{9}$	From $10\frac{3}{10}$	From $19\frac{5}{11}$
Take [illegible]	Take $8\frac{14}{27}$	Take $1\frac{7}{12}$	Take $0\frac{7}{15}$
Rem. $15\frac{11}{12}$	$4\frac{16}{27}$	Rem. $8\frac{43}{60}$	Rem. $18\frac{163}{165}$

	s.	d.		
From $\frac{7}{9}$ of a £. =	15	$6\frac{2}{3}$	From 7	weeks
Take $\frac{3}{10}$ of a s. =	0	$3\frac{1}{5}$	Take $9\frac{7}{10}$	days
Rem.	15	$3\frac{1}{15}$	Rem. 5*w*. 4*d*. 7*h*. 12*m*.	

SECTION 4.

Multiplication of Vulgar Fractions.

RULE.

Reduce the compound fractions to simple ones, and mixed numbers to improper fractions, then multiply the numerators together for a new numerator, and the denominators for a new denominator.

EXAMPLES.

1. Multiply $\frac{2}{3}$ by $\frac{1}{4}$

$$\frac{2\times 1}{3\times 4}=\frac{2}{12}=\frac{1}{6}\ \textit{Ans.}$$

2. Multiply $4\frac{1}{2}$ by $\frac{1}{8}$

$$\begin{array}{l} 4\frac{1}{2} \\ 2 \\ - \\ 9\times 1 = 9 \\ - \quad - \quad - \quad \textit{Ans.} \\ 2\times 8 = 16 \end{array}$$

3. Multiply $\frac{3}{8}$ by $\frac{4}{5}$ . . . . . *Ans.* $\frac{3}{10}$
4. $\frac{3}{5}$ of $\frac{3}{4}$ by $\frac{1}{2}$ . . . . . $\frac{9}{16}$
5. $7\frac{1}{2}$ by $\frac{1}{4}$ . . . . . $1\frac{7}{8}$
6. $\frac{3}{4}$ of 9 by $\frac{7}{8}$ . . . . . $5\frac{19}{32}$
7. $48\frac{3}{5}$ by $13\frac{4}{9}$ . . . . . $672\frac{3}{10}$

SECTION 5.

Division of Vulgar Fractions.

RULE.

PREPARE the fractions, if necessary, by reduction; invert the divisor, and proceed as in multiplication.

EXAMPLES.

1. Divide $\frac{3}{8}$ by $\frac{2}{3}$

$$\begin{array}{l} 3\times 3 \quad 9 \\ - \quad -=- \quad \textit{Ans.} \\ 8\times 2 \quad 16 \end{array}$$

2. Divide $4\frac{1}{2}$ by $1\frac{2}{3}$

$$\begin{array}{ll} 4\frac{1}{2} & 1\frac{2}{3} \\ 2 & 3 \\ - & - \\ 9 & 5 \\ - & - \\ 2 & 3 \end{array} \qquad \text{Then } \begin{array}{l} 9\times 3 \quad 27 \\ - \quad -=-=2\frac{7}{10} \quad \textit{Ans.} \\ 2\times 5 \quad 10 \end{array}$$

3. Divide $\frac{3}{8}$ by $\frac{6}{7}$ *Ans.* $\frac{21}{48}$
4. Divide $\frac{17}{21}$ by $\frac{3}{5}$ $1\frac{22}{63}$
5. Divide $1\frac{1}{2}$ by $4\frac{3}{16}$ $\frac{6}{16}$
6. Divide $\frac{7}{8}$ by 4 $\frac{7}{32}$
7. Divide $9\frac{1}{6}$ by $\frac{1}{2}$ of 7 $2\frac{13}{21}$
8. Divide $5205\frac{1}{5}$ by $\frac{4}{5}$ of 91 $71\frac{1}{2}$

SECTION 6.

The Rule of Three in Vulgar Fractions.

The operation of the Rule of Three in Vulgar Fractions, whether *direct*, *inverse*, or *compound*, is performed in the same manner and agreeably to the principles laid down in whole numbers under these rules.

When the question is in direct proportion, prepare the terms by reduction, and invert the first term; then proceed as in multiplication of fractions.

EXAMPLE.

1. If $\frac{1}{4}$ of a yard of cloth cost $\frac{2}{3}$ of a dollar, what will $\frac{7}{8}$ of a yard come to?

$$\begin{array}{lccccc} & yd. & & D. & & yd. \\ \text{As} & \frac{1}{4} & : & \frac{2}{3} & :: & \frac{7}{8} \\ & 4\times2\times7=56 & & & & \\ \text{Inverted} & - & - & - & - & =2\frac{1}{3} \text{ dollars.} \\ & 1\times3\times8=24 & & & & \end{array}$$

D. c.
Ans. 2 $33\frac{1}{3}$

2. If $\frac{2}{3}$ of a ton of iron cost $164\frac{1}{3}$ dollars, what will $\frac{6}{7}$ of a ton come to? *Ans.* 211 dolls. $28\frac{4}{7}$ cts.

3. A person having $\frac{3}{5}$ of a coal mine, sells $\frac{3}{4}$ of his share for 171 dollars, what is the value of the whole mine at the same rate? *Ans.* 380 dollars.

4. At $\frac{5}{6}$ of a dollar per yard, what will 42 yards come to? *Ans.* 35 dollars.

5. A gentleman owning $\frac{2}{5}$ of a vessel, sells $\frac{2}{3}$ of his share for 312 dollars, what is the whole vessel worth? *Ans.* 1170 dollars.

6. If $1\frac{1}{3}$ bushel of apples cost $79\frac{1}{3}$ cents, what will $3\frac{1}{5}$ bushels cost at the same rate? *Ans.* $202\frac{3}{10}$ cents.

7. If $\frac{1}{8}$ of a ship be worth 175 dollars 35 cents, what part of her may be purchased for 601 dollars 20 cents?

Ans. $\frac{3}{7}$

SECTION 7.

INVERSE PROPORTION.

RULE.

PREPARE the question as in direct proportion, invert the third term, and proceed as in multiplication of fractions.

EXAMPLES.

1. How much shalloon $\frac{3}{4}$ yard wide, will line $4\frac{1}{2}$ yards of cloth $1\frac{1}{2}$ yard wide?

$1\frac{1}{2}=\frac{3}{2}$
$4\frac{1}{2}=\frac{9}{2}$

Then, as $\frac{3}{2} : \frac{9}{2} :: \frac{3}{4}$

Or, inverted $\frac{3}{2} : \frac{9}{2} :: \frac{4}{3} = \frac{108}{12} = 9$ *Ans.*

2. If $6\frac{1}{2}$ hundred weight be carried $22\frac{9}{20}$ miles for $25\frac{3}{7}$ dollars, how far may 1 hundred weight be carried for the same money? *Ans.* $145\frac{1}{4}$ miles.

3. If 12 men can finish a piece of work in $37\frac{3}{5}$ days; how long will it take 16 men to do the same work?

Ans. $28\frac{1}{5}$ days.

4. A lends to B $100\frac{3}{4}$ dollars for $6\frac{3}{4}$ months; what sum should B lend to A for $3\frac{5}{6}$ years, to requite his kindness?

Ans. $14\frac{1}{2}\frac{2}{0}\frac{3}{7}$ dollars.

5. How many feet long must a board be, that is $\frac{7}{8}$ of a foot wide, to equal one that is $20\frac{1}{2}$ feet long, and $\frac{3}{4}$ of a foot wide? *Ans.* $17\frac{1}{4}$ feet long.

6. In exchanging $20\frac{1}{2}$ yards of cloth of $1\frac{1}{4}$ yard wide, for some of the same quality of $\frac{3}{4}$ yard wide, what quantity of the latter makes an equal barter?

Ans. $34\frac{1}{6}$ yards.

PART VII.

EXTRACTION OF THE ROOTS, AND COMPARATIVE ARITHMETIC.

SECTION 1.

Involution, or the Raising of Powers.

INVOLUTION is the multiplying of a given number by itself continually, any certain number of times.

The product of any number so multiplied into itself, is termed the power of that number.

Thus $2\times2=4=$ the second power or square of 2.
$2\times2\times2=8=$ the third power or cube of 2.
$2\times2\times2\times2=16=$ the fourth power of 2, &c.

The number denoting the power to which any given sum is raised, is called the *index* or *exponent* of that power.

If two or more powers are multiplied together, their product will be that power, whose index is the sum of the exponents of the factors. Thus $2\times2=4$, the 2d power of 2; $4\times4=16$, the 4th power of 2; $16\times16=256$, the 8th power of 2, &c.

TABLE

Of the first nine powers.

Roots	Squares	Cubes	4th power	5th power	6th power	7th power	8th power	9th power
1	1	1	1	1	1	1	1	1
2	4	8	16	32	64	128	256	512
3	9	27	81	243	729	2187	6561	19683
4	16	64	256	1024	4096	16384	65536	262144
5	25	125	625	3125	15625	78125	390625	1953125
6	36	216	1296	7776	46656	279936	1679616	10077696
7	49	343	2401	16807	117649	823543	5764801	40353607
8	64	512	4096	32768	262114	2097152	16777216	134217728
9	81	729	6561	59049	531441	4782969	43046721	387420489

EXAMPLE.

1. What is the 3d power of 15?

$15 \times 15 \times 15 = 3375$ *Ans.*

2 What is the 4th power of 35? *Ans.* 1500625.

3. What is the third power of 1,03? *Ans.* 1,092727.

4. What is the 5th power of ,029?

Ans. ,000000707281.

5. What is the 4th power of $\frac{3}{4}$? *Ans.* $\frac{81}{256}$

SECTION 2.

Of Evolution, or the Extracting of Roots.

EVOLUTION is the reverse of involution. For as $3 \times 3 = 9 \times 3 = 27$, the power; so $27 \div 3 = 9 \div 3 = 3$, the root of that power. Hence the root of any number, or power, is such a number as being multiplied into itself a certain number of times, will produce that power. Thus, 4 is the square root of 16, for $4 \times 4 = 16$; and 5 is the cube root of 125, for $5 \times 5 \times 5 = 125$.

SECTION 3.

THE SQUARE ROOT.

ANY number multiplied once into itself is called the square of that number. Hence, to extract the square root of any number, is to find such a number as being multiplied by itself, will be equal to the given number.

RULE.

1. Point off the given sum into periods of two figures *each*, beginning at the right hand.

2. Subtract from the first period on the left, the greatest square contained therein; setting the root, so found, for the first quotient figure.

3. Double the quotient for a new divisor, and bring down the next period to the remainder for a new dividual. Try how often the divisor is contained in the dividual, omitting the units figure, and place the number, so found, in the quotient, and on the right of the divisor; multiply and subtract as in division.

4. Double the quotient for a new divisor; bring down the next period, and proceed as before, till all the periods are brought down. When a remainder occurs, add ciphers for a new period, the quotient figure of which will be a decimal, which may be extended to any required degree of exactness.

PROOF.

Square the root, adding the remainder (if any) to the product, which will equal the given number.

EXAMPLE.

1. What is the square root of 531441?

```
                                  531441  (729  Ans.
                                  49
                                  ----
1. double the quotient 14,2)       414
                                   284
                                  -----
2. double      do.     144,9)     13041
                                  13041
                            729
                            729
                           ----
                           6561
                          1458
                         5103
                         ------
                         531441 proof.
```

2. What is the square root of 106929? *Ans.* 327
3. What is the square root of 4782969? 2187.
4. What is the square root of 43046721? 6561.
5. What is the square root of 387420489? 19683.
6. What is the square root of 22071204? 4698.
7. What is the square root of 36372961? 6031.
8. What is the square root of 2268741? 1506,23+
9. What is the square root of 7596796? 2756,228+

When there are decimals joined to the whole numbers in the given sum, make the number of decimals even by adding ciphers, and point off both ways, beginning at the decimal point.

10. What is the square root of 9712,718051 ?
Ans. 98,553+
11. What is the square root of 3,1721812 ?
Ans. 1,78106+
12. What is the square root of 4795,25731 ?
Ans. 69,247
13. What is the square root of ,00008836 ?
Ans. ,0094

To extract the square root of a vulgar fraction.

RULE.

Reduce the fraction to its lowest term; then extract the square root of the numerator for a new numerator, and the square root of the denominator for a new denominator.

Note. When the fraction is a *surd*, that is, a number whose exact root cannot be found, reduce it to a decimal and extract the root therefrom.

EXAMPLES.

1. What is the square root of $\frac{2304}{5184}$? *Ans.* $\frac{2}{3}$
2. What is the square root of $\frac{2704}{4225}$? $\frac{4}{5}$
3. What is the square root of $\frac{15625}{46656}$? $\frac{125}{216}$

Surds.

4. What is the square root of $\frac{357}{470}$? *Ans.* ,86602+
5. What is the square root of $\frac{478}{540}$? ,93309+
6. What is the square root of $\frac{337}{738}$? ,72414+

To extract the square root of a mixed number.

RULE.

1. Reduce the fractional part of the mixed number to its lowest term, and the mixed number to an improper fraction.

2. Extract the roots of the numerator and denominator, for a new numerator and denominator.

If the mixed number given be a surd, reduce the fractional part to a decimal, annex it to the whole number, and extract the square root therefrom.

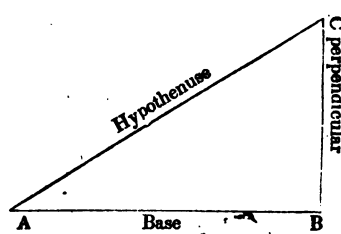
EXAMPLES.

1. What is the square root of $37\frac{2}{4}\frac{5}{9}$? *Ans.* $6\frac{1}{7}$
2. What is the square root of $27\frac{9}{16}$? $5\frac{1}{4}$
3. What is the square root of $51\frac{1}{2}\frac{1}{5}$? $7\frac{1}{5}$
4. What is the square root of $9\frac{3}{4}\frac{3}{9}$? $3\frac{1}{7}$

Surds.

5. What is the square root of $7\frac{9}{11}$? *Ans.* 2,7961+
6. What is the square root of $8\frac{5}{7}$? 2,9519+
7. What is the square root of $85\frac{1}{16}$? 9,27+

Any two sides of a right angled triangle given to find the third side.



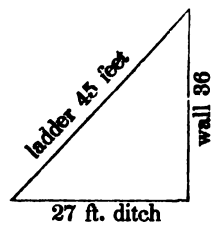
RULE.

As the square of the hypothenuse or longest side, is always equal to the square of the base and perpendicular, the other sides added together; then it is plain if the length of the two shortest sides are given, the square root of both these squared and added together, will be the length of the third or longest side.

Again, when the hypothenuse, or longest side, and one of the others are given; the square root of the difference of the squares of these two given sides will be the length of the remaining side.

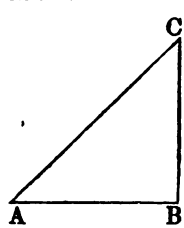
EXAMPLE.

1. The wall of a fortress is 36 feet high, and the ditch before it is 27 feet wide: it is required to find the length of a ladder that will reach to the top of the wall from the opposite side of the ditch? *Ans.* 45 feet.



2. The top of a castle from the ground is 45 yards high, and is surrounded with a ditch 60 yards broad, what length must a cable be to reach from the outside of the ditch to the top of the castle? *Ans.* 75 yards.

3. In a right angled triangle, A B C, the hypothenuse line A C is 45 feet, the base A B 27 feet; required the length of the perpendicular line B C? *Ans.* 36 feet.



4. In a right angled triangle, A B C, the line A C is 75 feet, B C 45 feet; required the length of the line A B? *Ans.* 60 feet.

To find the side of a square equal in area to any given superfices.

RULE.

Extract the square root of the content of the given superfices; the quotient will give the side of the equal square *sought.*

EXAMPLES.

1. If the content of a given circle be 160, what is the side of the square equal? *Ans.* 12,64911+

2. If the area of a circle be 2025, what is the side of the square equal? *Ans.* 45.

3. If the area of a circle be 750, what is the side of the square equal? *Ans.* 27,38612+

To find the diameter of a circle of a given proportion larger or less than a given one.

RULE.

Square the diameter of the given circle, and multiply (if greater) or divide (if less) the product, by the number of times the required circle is greater or less than the given one.

EXAMPLES.

1. There is a circle whose diameter is 4 feet; what is the diameter of one 4 times as large? *Ans.* 8 feet.

2. A has a circular yard of 100 feet diameter, but wishes to enlarge it to one of 3 times that area; what will the diameter of the enlarged one measure? *Ans.* 173,2+

3. If the diameter of a circle be 12 inches, what will be the diameter of another circle of half the size?

Ans. 8,48+inches.

The area of a circle given to find the diameter.

RULE.

Multiply the square root of the area by 1,12837, and the produce will be the diameter.

EXAMPLES.

1. When the area is 160, what is the diameter?

Ans. 14,272947+

2. What length of a halter will be sufficient to fasten a horse from a post in the centre, so that he may be able to graze upon an acre of grass, and no more?

Ans. 7,1364 perches, or 117 ft. 9 inches.

Application.

1. If an army of 20736 men is formed into a square column; how many men will each front contain?

Ans. 144 men.

2. How many feet of boards will it require to lay the floor of a room that is 25 feet square? *Ans.* 625 feet.

3. A certain square pavement contains 191736 square stones, all of the same size; how many are contained in one of its sides? *Ans.* 444.

4. In a triangular piece of ground containing 600 perches, one of the shortest sides measures 40 perches, and the other 30; what is the length of the longest side?

Ans. 50 perches.

5. Two gentlemen set out from Pittsburgh at the same time; one of them travels 84 miles due north, and the other 50 miles due west; what distance are they asunder?

Ans. $97\frac{1}{2}+$ miles.

6. What is the square root of 964,5192360241?

Ans. 31,05671.

7. What is the square root of 1030892198,4001?

Ans. 32107,51.

As it is probable many teachers find it difficult to explain satisfactorily the reasons and principles upon which the rules for the extraction of the roots are founded, I have subjoined the following *demonstration* of the rule for extracting the square root; and which will also serve to show the reason of the rules for extracting the roots of the higher powers. From what has been already said on this rule, it is sufficiently evident that the extraction of the square root has always this operation on numbers, *viz. to arrange the number of which the root is extracted into a square form.* Thus, if a carpenter should have 625 feet of dressed boards for laying a floor; if he extracts the square root of this number, (625) he will have the exact length of one side of a square floor, which these boards will be sufficient to make.

Let this then be the question: Required to find the length of one side of a square room, of which 625 square feet of boards will be sufficient to lay the floor.

The first step, according to the rule given, *is to point off into periods of two figures each, beginning*

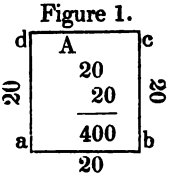
at the unit's place. This ascertains the number of figures of which the root will consist, from this principle, that the product of any two numbers can have, at most, but so many places of figures, as there are places in both the factors, and at least, but one less.

The number (625) will then have two periods, and consequently the root will consist of two figures.

Operation.

```
 . .
625 ( 2
 4
 ---
225
```

Figure 1.



The last, or left-hand period in this number is 6, in which 4 is the greatest square, and 2 the root; hence 2 is the first figure in the root, and as one figure more is yet to be found, we may for the present supply the place of that figure with a cipher (20); then 20 will express the just value of that part of the root now obtained. But a root is the side of a square, of equal sides. Hence, figure 1 exhibits a square, each side of which is 20 feet, and the area 400, of which 20 is the root now obtained.

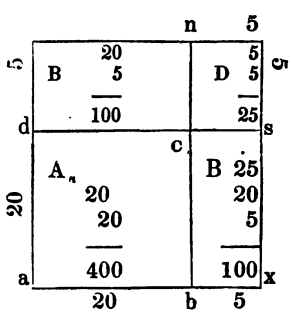
As the rule requires, we next *subtract the greatest square contained in the first period, and to the remainder bring down the next period.* 4 is the greatest square contained in the first period (6), and as it falls in the place of hundreds, is in reality 400, as may be seen by filling up the places to the right-hand with ciphers; this subtracted from the 6 leaves 2, as a remainder, to which if the next period is brought down, the remainder will be 225; and the original number of feet (625) has been diminished by the deduction of 400 feet, a number equal to the superficial content of the square A.

Figure 1, therefore, exhibits the exact progress of the operation, and shows plainly how 400 feet of the boards have been disposed in the operation thus far, and also that 225 feet yet remains to be added to this square, by enlarging it in such a manner as not to destroy its quadrate form, or its continuing a complete and perfect square. Should the addition be made to *one side*, only, the figure would lose its square. The addition must be made to *two sides*: accordingly the rule directs to "*double the quotie*

(*viz.* the root already found) *for a new divisor;*" the double of the root is equal to two sides of the square A, for the double of 2 is 4, and as this 4 falls in the place of tens, since the next figure in the root, according to the rule, is to be placed before it in the place of units, it is in reality 40, and equal to *a b* and *b c* which are 20 each.

Operation continued.

```
      . .
      625 ( 25
      4
      ---
  45 ) 225
       225
       ---
       000
```



Again, as the rule directs, *try how often the divisor is contained in the dividual, omitting the units figure.*

The divisor is here 4, which, as has already been shown, is 4 tens, or 40; this is to be divided into the remainder 225; omitting the last figure=220. But 40=the sum of the two sides *b c* and *c d;* to which the remaining 225 is to be added, and the square A enlarged, which omitting the last figure (5) gives 5 for the last quotient figure; 5 is the breadth of the two parallelograms B B, the area of each (5 × 20) is 100.

The rule requires us *to omit the last figure in the dividual*, and also, *to place the quotient figure, when found, on the right of the divisor*, the reasons for which are, that additions of the two parallelograms B B to the sides of the square A (fig. 2) do not leave it a perfect square, a deficiency remaining at the corner D; the right-hand figure is omitted to leave something of the dividend for this deficiency. And as this deficiency is limited by the two parallelograms B B, and the quotient figure (5) is the breadth of these, consequently the quotient 5=the length of each of the sides of the small square D; this quotient then being placed on the right of the divisor and multiplied into itself gives the area of the square D; which being added to the

contents of the two parallelograms B B each (100) 200, shows that the remaining 225 feet of boards have been disposed of, in these three additions (B B D) made to the first square A; whilst the figure is seen to be continued a complete square.

Q. E. D.

PROOF.

The square A	=400 feet
The parallelograms B B	=200
The square D	= 25
	625 feet.

SECTION 4.

THE CUBE ROOT.

THE cube is the third power of any number, and is found by multiplying that number twice into itself. As $2\times2\times2=8$.

To extract the cube root, therefore, of any number, is to find another number, the cube of which will equal the given number. Thus 4 is the cube root of 64; for $4\times4\times4=64$.

RULE.

1. Point off the given number into periods of three figures each, beginning at the units place, or decimal point. These periods will show the number of figures contained in the required root.

2. Find the greatest cube contained in the first period, and subtract it therefrom; put the root of this cube in the quotient, and bring down the next period to the remainder for a new dividual.

3. Square the quotient and multiply it by 3 for a defective divisor; $2\times2\times3=12$. Find how often this is contained in the dividual, rejecting the units and tens therein, and *place the result in* the quotient, and its square to the right of *the divisor.* $4\times4=16$ put to the divisor $12=1216$

4. Multiply the last figure in the quotient by the rest, and the product by 30; add this to the defective divisor, and multiply this sum by the last figure in the quotient, subtract that product from the dividual, bring down the next period, and proceed as before.

Note. When the quotient is 1, 2, or 3, put a cipher in the place of tens in filling up the square on the right of the divisor.

EXAMPLE.

1. What is the cube root of 48228544?

Operation.

48228544 (364 *Ans.*

Greatest cube in 48 is 27

21228

Square of 3 × by 3=27. 1 def. divis.
Square of 6 put to 27 =2736
6 last quo. fig. × by the rest and 30 = 540
Complete divisor 3276

19656

1572544

Square of 36 × 3=3888. 2 def. divis.
Square of 4 put to 3888=388816
4 last quo. fig. × by the rest and 30 = 4320
Complete divisor 393136

1572544

2. What is the cube root of 13824? *Ans.* 24
3. What is the cube root of 373248? 72
4. What is the cube root of 5735339? 179
5. What is the cube root of 84604519? 439
6. What is the cube root of 27054036008? 3002
7. What is the cube root of 122615327232? 4968
8. What is the cube root of 22069810125? 2805
9. What is the cube root of 219365327791? 6031
10. What is the cube root of 673373097125? 8765
11. What is the cube root of 12,977875? 2,35
12. What is the cube root of 15926,972504? 25,16+
13. What is the cube root of 36155,027576? 33,06+

14. What is the cube root of ,053258279? *Ans.* ,376+
15. What is the cube root of ,001906624? ,124
16. What is the cube root of ,000000729? ,009
17. What is the cube root of 2? 1,25+

To extract the cube root of a vulgar fraction.

RULE.

Reduce the fraction to its lowest terms; then extract the cube root of the numerator for a new numerator, and the cube root of the denominator for a new denominator; but if the fraction be a surd, reduce it to a decimal, and extract the root from it for the answer.

EXAMPLES.

1. What is the cube root of $\frac{2}{6}\frac{5}{4}\frac{0}{6}$? *Ans.* $\frac{5}{7}$
2. What is the cube root of $\frac{3}{1}\frac{2}{5}\frac{4}{0}\frac{}{0}$? $\frac{3}{5}$
3. What is the cube root of $\frac{1}{5}\frac{5}{1}\frac{2}{3}\frac{0}{0}$? $\frac{2}{3}$

Surds.

4. What is the cube root of $\frac{4}{7}$? *Ans.* ,829+
5. What is the cube root of $\frac{2}{3}$? ,873+
6. What is the cube root of $\frac{5}{9}$? ,822+

To extract the cube root of a mixed number.

RULE.

Reduce the fractional part to its lowest terms, and the mixed number to an improper fraction; extract the cube roots of the numerator and denominator for a new numerator and denominator; but if the mixed number given be a surd, reduce the fractional part to a decimal, annex it to the whole number, and extract the root therefrom.

EXAMPLES.

1. What is the cube root of $31\frac{15}{34}$? Ans. $3\frac{1}{4}$
2. *What is the* cube root of $12\frac{19}{21}$? $2\frac{3}{5}$
3. *What is the* cube root of $405\frac{29}{125}$? $7\frac{2}{5}$

Surds.

4. What is the cube root of $7\frac{1}{3}$? *Ans.* 1,93+
5. What is the cube root of $8\frac{4}{7}$? 2,057+
6. What is the cube root of $9\frac{1}{8}$? 2,092+

To find the side of a cube that shall be equal to any given solid, as a globe, a cone, &c.

RULE.

Extract the cube root of the solid content of any solid body, for the side of the cube of equal solidity.

EXAMPLES.

1. If the solid content of a globe is 10648, what is the side of a cube of equal solidity? *Ans.* 22.
2. If the solid content of a globe is 389017, what is the side of a cube of equal solidity? *Ans.* 73.

Note. The relative size of different cubical vessels is found by multiplying the cube of the side of the given vessel, by the proportional number, and taking the cube root of the product for the answer sought.

EXAMPLES.

1. There is a cubical vessel whose side is two feet · I demand the size of another vessel which shall contain three times as much? *Ans.* 2 feet 10 inches and $\frac{1}{5}$ nearly.
2. There is a cubical vessel whose side is 1 foot; required the side of another vessel that shall contain three times as much? *Ans.* 17,306 inches

Application.

1. If a ball of 6 inches diameter weigh 32 lb., what will one of the same metal weigh, whose diameter is 3 inches? *Ans.* 4 lb.
2. What is the side of a cubical mound equal to one 288 feet long, 216 broad, and 48 high? *Ans.* 144 feet.
3. There is a stone of cubic form, which contains 389017 *solid feet;* what is the superficial content of one of its sides? Ans. 5329 feet.

4. What is the difference between half a solid foot, and a solid half foot? *Ans.* 3 half feet.

5. In a cubical foot, how many cubes of 6 inches, and how many of 4 are contained therein?

Ans. 8 of 6 inches, and 27 of 4 inches.

SECTION 5.

OF PROGRESSION.

PROGRESSION is of two kinds, arithmetical and geometrical.

Arithmetical progression is when any series of numbers increase or decrease regularly by a common difference. As 1, 2, 3, 4, 5, 6, &c. are in arithmetical progression by the continual adding of one; and 9, 7, 5, 3, 1, by the continual subtracting of two.

Note.—In any series of even numbers in arithmetical progression, the sum of the two extremes will be equal to the sum of any two terms equally distant therefrom; as 2. 4. 6. 8. 10. 12., where $2+12=14$, so $4+10=14$, and $6+8=14$. But if the number of terms is odd, the double of the middle term will be equal to any two of the terms equally distant therefrom; as 3. 6. 9. 12. 15. where the double of 9 the middle term $=18$, and $3+15=18$, or $6+12=18$.

In arithmetical progression five things must be carefully observed, *viz.*

1. The first term,
2. The last term,
3. The number of terms,
4. The equal difference,
5. The sum of all the terms.

Case 1.

The first term, common difference, and number of terms, given to find the last term, and sum of all the terms.

RULE.

***1. Multiply* the number of terms, less 1, by the common**

difference, and to the product add the first term, the sum will be the last term.

2. Multiply the sum of the two extremes by the number of terms, and half the product will be the sum of all the terms.

EXAMPLE.

1. A merchant bought 50 yards of linen, at 2 cents for the first yard, 4 for the second, 6 for the third, &c. increasing two cents every yard; what was the price of the last yard, how much the whole amount, and what the average price per yard?

50 number of terms
1
—
49 number of terms less one
Multiply by 2 common difference
—
98
Add 2 first term
—
100 last term
—
2+100=102 sum of the two extremes
Multiply by 50 number of terms
—
2)5100
—
50)25,50 sum of all the terms
—
51 cents

Ans. { 100 cents the last yard
25,50 do. the whole amount
51 do. the average price per yd.

2. Bought 20 yards of calico at 3 cents for the first yard, 6 for the second, 9 for the third, &c.; what did the whole cost? *Ans.* 6 dolls. 30 cents.

3. If 100 apples were laid two yards distant from each other, in a right line, and a basket placed two yards distant from the first apple, what distance must a person travel to gather them singly into the basket?

Ans. 11 miles, 3 furlongs, 180 yards.

4. A agreed to serve B 10 years, at the rate of 20 dollars for the first year, 30 for the second, 40 for the third, &c.; what had he the last year, how much for the whole time, and what per annum?

Ans. 110 dolls. for the last year, 650 dolls. the whole amount, and 65 dolls. per annum.

5. A sold to B 1000 acres of land, at 10 cents for the first acre, 20 for the second, 30 for the third, &c.; what was the price of the last acre, and what did the whole come to?

Ans. { 100 dolls. the last acre,
50050 do. whole cost.

Case 2.

When the two extremes, and number of terms are given, to find the common difference.

RULE.

Divide the difference of the extremes, by the number of terms, less one; the quotient will be the common difference.

EXAMPLE.

A is to receive from B a certain sum to be paid in 11 several payments in arithmetical progression; the first payment to be 20 dollars, and the last to be 100 dollars; what is the common difference, what was each payment, and how much the whole debt?

Operation.

100 last term
20 first term

No. of terms $11-1=10$) 80 the difference

8 common difference

$20+100\times 5\frac{1}{2}=660$ whole debt
20 first payment
$20+8=28$ second do.
$28+8=36$ third do. &c.

2. There are 21 persons whose ages are equally distant *from each other*, in arithmetical progression; the youngest

is 20 years old, and the eldest 60; what is the common difference of their ages, and the age of each man?

Ans. { 2 common difference
20+2=22 the second
22+2=24 the third, &c.

3. A man is to travel from Pittsburgh to a certain place in 12 days, and to go but three miles the first day, increasing each day's journey in arithmetical progression, making the last day's travelling 58 miles; what is the daily increase, and what the whole distance?

Ans. { 5 miles daily increase
366 miles whole distance.

SECTION 6.

GEOMETRICAL PROGRESSION.

Any series of numbers increasing or decreasing by one continual multiplier, or divisor, called the *ratio*, is termed *geometrical progression;* as 2, 4, 8, 16, 32, &c. increase by the multiplier 2; and 32, 16, 8, 4, 2, decrease, continually, by the divisor 2.

In geometrical progression there are five things to be carefully observed.

1. The first term,
2. The last term,
3. The number of terms,
4. The ratio,
5. The sum of all the terms.

To find the last term, and sum of all the series in geometrical progression, work by the following

RULE.

1. Raise the ratio in the given sum, to that power whose index shall always be one less than the number of terms *given; multiply* the number so found by the first term, and *the product will* be the last term, or greater extreme.

2. Multiply the last term by the ratio, from that product subtract the first term, and divide the remainder by the ratio, less one; the amount will be the sum of the series, or of all the terms.

EXAMPLE.

1. Suppose 20 yards of broadcloth was sold at 4 mills for the first yard, 12 for the second, 36 for the third, &c. what did the cloth come to, and what was gained by the sale, supposing the prime cost to have been $15 per yard?

Note. In this question observe, the first term is 4, the ratio 3, and the number of terms 20, consequently the ratio 3 must be raised to 20—1=19th power. Thus,

	1st power	2d power	3d power	4th power
Indices	1	2	3	4
Ratio	3	9	27	81
				81
				81
				648
				6561=the 8th power
				6561
				6561
			39366	
			32805	
			39366	
			43046721=16th power	
			27=3d power	
			301327047	
			86093442	
			1162261467=19th power	
			×4 first term	
			4649045868=20th or last term	
			×3	
			13947137604	
			—4 first term	
Ratio 3—1=2)			13947137600	
			6973568,80=sum of the series, or number of	
First cost of the cloth			300,00 mills for which the cloth was sold	
			6973268,80=gain.	

N

2. A father gave his daughter who was married on the first day of January, one dollar towards her portion, promising to double it on the first day of every month for one year; what was the amount of her whole portion?

Ans. 4095 dollars.

3. A merchant sold 15 yards of satin; the first yard for 1*s.* the second for 2*s.* the third for 4*s.* &c. in geometrical progression; what was the price of the 15 yards?

Ans. 1638*l.* 7*s.*

4. A goldsmith sold 1 pound of gold at 1 cent for the first ounce, 4 for the second, 16 for the third, &c.; what did it come to, and what did he gain, supposing he gave 20 dollars per ounce?

Ans. He sold it for 55924 dollars 5 cents, and gained 55684 dollars 5 cents.

5. What sum would purchase a horse with 4 shoes and 8 nails in each shoe, at one mill for the first nail, 2 mills for the second, 4 for the third, &c. doubling in geometrical progression to the last?

Ans. 4294967 dollars 29 cents 5 mills.

6. What sum would purchase the same horse, with the same number of shoes, and nails, at 1 mill for the first nail, 3 for the second, 9 for the third, &c., in a triple ratio of geometrical progression to the last?

Ans. 926510094425 dollars 92 cents.

7. What sum would purchase the same horse, with the same number of shoes, and nails, at 1 mill for the first nail, 4 for the second, 16 for the 3d, &c., in a quadruple ratio of geometrical progression to the last?

Ans. 6148914691236517 dollars 20 cents 5 mills.

8. Sold 30 yards of silk velvet, at 2 pins for the first yard, 6 for the second, 18 for the third, &c., and these disposed of at 1000 for a farthing; what did the velvet amount to, and what was gained by the sale, supposing the prime cost to have been 100*l.* per yard?

Ans. { Amount 214469929*l.* 5*s.* 3½*d.*
{ Gained 214466929*l.* 5*s.* 3½*d.*

SECTION 7.

OF POSITION.

POSITION is a rule for finding the true number, by one or more false or supposed numbers, taken at pleasure.

It is of two kinds, *viz. Single* and *Double.*

Single position teaches to resolve such questions as require but one supposed number.

RULE.

1. Suppose any number whatever, and work in the same manner with it as is required to be performed in the given question.

2. Then, as the amount of the errors, is to the supposed sum, so is the given number to the one required.

PROOF.

Add the several parts of the result together, and if it agrees with the given sum, it is right.

EXAMPLES.

1. A schoolmaster being asked how many scholars he had, said, if I had as many, half as many, and one quarter as many more, I should have 132; how many had he?

Suppose he had	40
as many	40
½ as many	20
¼ as many	10
	110

As 110 : 40 :: 132 : 48 *Ans.*

Proof	48
	48
	24
	12
	132

2. It is required to divide a certain sum of money among 4 persons, in such a manner that the first shall have ⅓, the second ¼, the third ⅛, and the fourth the remainder, which is 28 dollars; what was the sum?

Suppose	72
⅓ is	24
¼ is	18
⅛ is	12
	54
Rem.	18

As 18 : 72 :: 28 : 112 dolls. *Ans.*

Proof	112
⅓ is	37⅓
¼ is	28
⅛ is	18⅔
	84
	28 last share.

3. A, B, and C, buy a carriage for 340 dollars, of which A pays three times as much as B, and B four times as much as C; what did each pay?

Ans. A paid 240 dolls.
B 80
C 20

4. What is the sum of which $\frac{1}{4}$, $\frac{1}{5}$, and $\frac{1}{6}$, make 148 dollars? *Ans.* 240 dollars.

5. A person having spent $\frac{1}{2}$, and $\frac{1}{3}$ of his money, had $26\frac{2}{3}$ dollars left; what had he at first? *Ans.* 160 dollars.

6. A, B, and C, talking of their ages, B said his age was once and a half the age of A; C said his was twice and $\frac{1}{10}$ the age of both, and that the sum of their ages was 93; what was the age of each?

Ans. A's age 12 years.
B's 18
C's 63

7. Seven-eighths of a certain number exceeds four-fifths by 6; what is that number? *Ans.* 80.

8. A gentleman bought a chaise, horse, and harness, for 360 dollars; the horse came to twice the price of the harness, and the chaise to twice the price of the horse and harness together; what did he give for each?

Ans. 80 dollars for the horse
40 harness
240 chaise

9. A gentleman being asked the price of his carriage, answered that $\frac{1}{3}$, $\frac{1}{4}$, $\frac{1}{5}$, and $\frac{1}{6}$ of its price was 228 dollars; what was the price of the carriage? *Ans.* 240 dolls.

10. A saves $\frac{1}{3}$ of his wages, but B, who has the same salary, by spending twice as much as A, sinks 50 dollars a year; what is their annual salary? *Ans.* 150 dolls. each.

SECTION 8.

DOUBLE POSITION.

Double Position is making use of two supposed numbers, to find the true one.

RULE.

1. Take any two numbers, and proceed with them according to the conditions of the question, noting the errors

of the results; multiply these errors cross-wise, *viz.* the first position by the last error, and the last position by the first error.

2. If the errors be alike, that is, both greater, or both less than the given number, take their difference for a divisor, and the difference of the products for a dividend: but if they are unlike, take their sum for a divisor, and the sum of the products for a dividend, the quotient will be the answer required.

EXAMPLE.

1. A father leaves his estate to be divided among his three sons, A, B, and C, in the following manner, *viz.* A is to have one-half wanting 50 dollars, B one-third, and C 10 dollars less than B; what was the sum left, and what was each son's share?

Operation.

1st. Suppose 240 dollars.

Then 240 ÷ 2—50 = 70 A's part
240 ÷ 3 = 80 B's part
B's share 80—10 = 70 C's part

Sum of all their parts 220

20 er. too little.

2d. Suppose 300 dollars.

Then 300 ÷ 2—50 = 100 A's part
300 ÷ 3 = 100 B's part
B's share 100—10 = 90 C's part

Sum of all their parts 290

10 er. too little.

errors.

1st. sup. 240 × 20 = 6000
2d. sup. 300 × 10 = 2400

10) 3600 (360 *Ans.*

Proof 360 ÷ 2—50 = 130
360 ÷ 3 = 120
120—10 = 110

360

2. A and B have the same income; A saves the $\frac{1}{8}$ of his, but B by spending 30 dollars per annum more than A, at the end of 8 years finds himself 40 dollars in debt; what is their income, and how much does each spend per annum?

Ans. { Their income is 200 dolls. per annum
A spends 175
B spends 205

3. A, B, and C, would divide 100 dollars between them, so as B may have 3 dollars more than A, and C 4 dollars more than B; how many dollars must each have?

Ans. { A 30 dollars.
B 33
C 37

4. A, B, and C, built a house which cost 10,000 dollars; A paid a certain sum, B paid 1000 dollars more than A, and C paid as much as both A and B; how much did each one pay?

Ans. { A paid 2000 dolls.
B 3000
C 5000

5. A gentleman has 2 horses and a saddle worth 50 dollars, which saddle if he put on the back of the first horse, will make his value double that of the second; but if he put it on the second horse, it will make his value triple that of the first; what is the value of each horse?

Ans. { First horse 30 dolls.
Second do. 40

6. The head of a fish is 9 inches long, and its tail is as long as its head and half its body, and its body is as long as its head and tail together; what is its whole length?

Ans. 6 feet.

7. A laborer hired 40 days upon this condition, that he should receive 20 cents for every day he wrought, and forfeit 10 cents for every day he was idle; at settlement he received 5 dollars; how many days did he work, and how many was he idle? *Ans.* Wrought 30 days, idle 10

8. A and B vested equal sums in trade; A gained a sum equal to $\frac{1}{4}$ of his stock, and B lost 225 dollars; then A's money was double that of B's; what sum had each vested?

Ans. 600 dollars.

9. Divide 15 into two such parts, so that when the greater

is multiplied by 4, and the less by 16, the products will be equal? *Ans.* Greater 12, less 3.

10. A person being asked in the afternoon what o'clock it was, answered, that the time past from noon was equal to $\frac{2}{11}$ of the time to midnight; what o'clock was it?

Ans. 36 minutes past 1 o'clock.

SECTION 9.

PERMUTATION.

PERMUTATION is the finding how many different ways any given number of things may be varied in position, or arrangement. Thus,

1	2	3	are six different arrange-
1	3	2	ments made upon the three
2	1	3	figures 1 2 3.
2	3	1	
3	1	2	
3	2	1	

RULE.

Multiply all the terms from an unit up to the given number into one another, and the last product will be the number of changes required.

EXAMPLE.

1. In how many different positions may 5 persons be placed at a table.

$$1\times2\times3\times4\times5=120 \quad \textit{Ans.}$$

2 How many changes may be rung on 12 bells, and how long would they be ringing but once over, allowing 10 changes to be rung in 1 minute, and the year to contain 365 days 6 hours?

Ans. 479001600 changes, and would require 91 years, 3 weeks, 5 days, and 6 hours.

3. Seven men not agreeing with the owner of a boarding house about the price of boarding, offer to give 100 dollars each, for as long time as they can seat themselves every day differently at dinner; this offer being accepted, how long *may they stay? Ans.* 5040 days, or 13 years, 295 da

4. What number of variations will the 9 digits admit of?
Ans. 362880.

5. How many changes may be made on the 26 letters of the alphabet?

Ans. 403,291,461,126,605,635,584,000,000.

Quatril. Trilns. Billions. Millions. Units.

Note.—From the answer to this last question, which amounts to a number, of which we cannot form any conceivable idea, we may discover the surprising power of numbers, and also the endless variety of ideas that may be distinctly communicated by these 26 simple characters. It will also be evident from the method of notation here used, that a row of figures of any given length whatever, may be numerated, though we may be entirely unable to comprehend the amount.

SECTION 10.

COMBINATION.

COMBINATION of quantities, is the showing how often a less number of things can be taken out of a greater, and combined or joined together differently.

RULE.

Take a series of 1, 2, 3, &c. up to the number to be combined; take another series of as many places, decreasing by unity from the number out of which the combinations are to be made; multiply the first continually for a divisor, and the latter for a dividend, and the quotient will be the answer.

EXAMPLE.

1. How many combinations may be made of 7 dollars out of 12?

$1 \times 2 \times 3$, &c. up to $7 = 5040$ divisor.

Again, 12 the whole number of terms less $7 = 5$

Hence $12 \times 11 \times 10$, &c. down to $5 = 3991680$ dividend.

And 5040) 3991680 (792 *Ans.*

2. How many combinations can be made of 6 letters out of 24 of the alphabet? *Ans.* 134596.

3. In how many different ways may an officer select 8 30, so as not to make the same selection twice
Ans. 5852925

PART VIII.

MENSURATION.

SECTION 1.

Duodecimals, or Cross Multiplication.

DUODECIMALS are fractions of a foot, or of an inch, or parts of an inch, having 12 for their denominator. Inches and parts are sometimes called primes (′), seconds (″), thirds (‴), &c.

The denominations are,

12 Fourths (⁗)	make	1 Third	‴
12 Thirds	-	1 Second	″
12 Seconds	-	1 Inch	*in.*
12 Inches	-	1 Foot	*ft.*

Note.—This rule is much used in measuring and computing the dimensions of the several parts of buildings; it is likewise used to find the tonnage of ships, and the contents of bales, cases, boxes, &c.

ADDITION OF DUODECIMALS.

RULE.

Add as in compound addition, carrying 1 for each 12 to the next denomination.

EXAMPLE.

Ft.	*in.*	″	‴	⁗
25	9	3	5	8
34	3	9	2	7
28	10	4	8	4
64	11	9	7	2
82	7	5	6	8
15	3	7	9	10
44	6	11	2	8
22	3	6	1	5
318	*8*	*9*	*8*	*4*

Ft.	*in.*	″	‴	⁗
244	6	3	10	5
355	9	8	5	1
559	10	9	5	8
129	5	5	6	9
895	1	10	5	11
651	1	7	5	9
555	9	8	5	5
388	11	10	10	9

SUBTRACTION OF DUODECIMALS.

RULE.

Work as in compound subtraction, borrowing 12 when necessary.

EXAMPLE.

	Ft.	*in.*	′	″	‴		*Ft.*	*in.*	′	″	‴
From	125	4	3	8	2		2756	5	7	8	0
Take	68	9	2	10	1		1839	9	5	11	10
Rem.	56	7	0	10	1						

3. From a board measuring 35 feet, 9 inches, 2 seconds, cut 24 feet, 10 inches, 5 seconds, and 4 thirds; what is left?
Ans. 10*ft.* 10*in.* 8*sec.* 8‴

4. A joiner having lined several rooms very curiously with costly materials, finds the amount to be, in square measure, 803 feet, 3 inches, 4 seconds; but several deductions being to be made for windows, arches, &c. those deductions amounted to 70 feet, 3 inches, 7 seconds, 10 thirds, 5 fourths; how many feet of workmanship must he be paid for? *Ans.* 732*ft.* 11*in.* 8″ 1‴ 7⁗

MULTIPLICATION OF DUODECIMALS.

Case 1.

When the feet of the multiplier do not exceed 12.

RULE.

Set the feet, or the highest denomination of the multiplier under the lowest denomination of the multiplicand, and multiply as in compound numbers, carrying 1 for every 12 from one denomination to another, and place the result of the lowest denomination in the multiplicand under its multiplier.

TABLE.

1. Feet multiplied by feet, give feet.
2. Feet multiplied by inches, give inches.
   nultiplied by seconds, give seconds, &c.
   multiplied by inches, give seconds.

5. Inches multiplied by seconds, give thirds, &c.
6. Seconds multiplied by seconds, give fourths.
7. Seconds multiplied by thirds, give fifths, &c.

PROOF.

Reduce the given sum to a decimal, or work by the rules of practice.

EXAMPLES.

1. Multiply

	Ft.	*in.*	"		*Ft.*	*in.*
	8	6	9	by	7	3
			7	3		
	59	11	3			
	2	1	8	3		
Ans. *Ft.*	62	0	11	3		

Or practice.

3 is $|\frac{1}{4}|$ 8 6 9
7 3
59 11 3
2 1 8 3
62 0 11 3 *Ans.*

Proof decimally.

8 6 9 = 8,5625
7 3 = 7,25
428125
171250
599375
62,078125
12
0,937500
12
11,250000
12
3,000000

		Ft.	*in.*		*Ft.*	*in.*		*Ft.*	*in.*	"
2.	Multiply	9	5	by	3	11	*Ans.*	36	10	7
3.		7	10	by	8	11		69	10	2
4.		8	4 6	by	2	7 4		21	10	5

5. What is the price of a marble slab, whose length is 5 feet 7 inches, and breadth 1 foot 10 inches, at 1 dollar and 50 cents per foot? *Ans.* 15 dolls. $35\frac{5}{12}$ cts.

6. There is a house with three tiers of windows, 3 in a tier, the height of the first tier is 7 feet 10 inches, of the second 6 feet 8 inches, and of the third 5 feet 4 inches, and *the breadth of each* window is 3 ft. 11 inches.; what will the *glazing come to* at 14 cts. per ft.? Ans. 32 dolls. 62½ cts.

Case 2.

When the feet of the multiplier exceeds 12.

RULE.

Multiply by the feet in the multiplier, and take parts for the inches.

EXAMPLES.

1. Multiply 84 feet 6 inches, by 36 feet 7 inches and 6 seconds.

Operation.

84 6

6 × 6 = 36

507 0

6

6*in.*	½	3042	0		
1	1/6	42	3		
6*sec.*	½	7			
		3	6		

Ans. 3094 9

	Ft.	*in.*		*Ft.*	*in.*		*Ft.*	*in.*	*sec.*
2. Multiply	76	7	by	19	10	*Ans.*	1518	10	10
3.	127	6	by	92	4		11772	6	
4.	184	8	by	127	6		23545		

	Ft.	*in.*	*sec.*		*Ft.*	*in.*	*sec.*
5. Multiply	311	4	7	by	36	7	5

6 × 6 = 36

1868 3 6

6

6*in.* is	½	11209	9	0	′′′	
1	1/6	155	8	3	6	
4*sec.*	⅓	25	9	8	7	′′′′
1	¼	8	7	2	10	4
		2	1	9	8	7

11402 0 0 7 11 Ans.

6. A floor is 70 feet 8 inches, by 38 feet 11 inches; how many square feet are therein?

Ans. 2750*ft.* 1*in.* 4*sec.*

7. If a ceiling be 59 feet 9 inches long, and 24 feet 6 inches broad, how many yards does it contain?

Ans. 162*yds.* 5*ft.* 10½*in.*

Note. Divide the square feet by 9, and the quotient will be square yards.

8. What will the paving of a court yard come to at 15 cents per yard, the length being 58 feet 6 inches, and the breadth 54 feet 9 inches?

Ans. 53 dolls. 38+cts.

9. What is the solid content of a bale of goods, measuring in length 7 feet 6 inches, breadth 3 feet 3 inches, and depth 1 foot 10 inches?

Ans. 44*ft.* 8*in.* 3*sec.*

Note. To find the cubic feet, or solid content of bales, cases, boxes, &c. multiply the length by the breadth, and that product by the thickness.

10. A merchant imports from London six bales of the following dimensions, *viz.*

	Length.		Breadth.		Depth.	
	Ft.	*in.*	*Ft.*	*in.*	*Ft.*	*in*
No. 1.	2	10	2	4	1	9
2.	2	10	2	6	1	3
3.	3	6	2	2	1	8
4.	2	10	2	8	1	9
5.	2	10	2	6	1	9
6.	2	11	2	8	1	8

What are the solid contents, and how much will the freight amount to, at 20 dollars per ton of 40 feet?

Ans. 71*ft.* 7*in.* and freight 35 dollars 79 cts.

To find a ship's tonnage by Carpenter's measure.

For single decked vessels, multiply the length, breadth at the main beam, and depth of the hold together, and divide the product by 95; but if the vessel be double decked, *take half the breadth* of the main beam for the depth of the *hold, and work as* for a single decked vessel.

EXAMPLES.

1. The length of a single decked vessel is 60 feet, the breadth 20, and depth 10; what is the tonnage?

Then $60 \times 20 \times 10 = 12000$

And $12000 \div 95 = 126\frac{6}{19}$ tons. *Ans.*

Or, as $95 : 20 \times 10 :: 60 : 126\frac{6}{19}$ *Ans.*

2. Required the tonnage of a double decked vessel, whose length is 90, and breadth 30.

Then $90 \times 30 \times 15$ (half breadth) $= 40500$.

And $40500 \div 95 = 426\frac{6}{19}$ tons. *Ans.*

Or, as $95 : 30 \times 15 :: 90 : 426\frac{6}{19}$ *Ans.*

3. A single decked vessel is 64 feet long, 22 feet broad, and 10 feet deep; what is its tonnage?

Ans. $148\frac{4}{19}$ tons.

4. What will be the tonnage of a double decked vessel whose length is 80 feet, and breadth 26 feet?

Ans. $284\frac{4}{19}$ tons.

To find the Government tonnage.

"If the vessel be double decked, take the length thereof from the fore part of the main stem, to the after part of the stern post, above the upper deck; the breadth thereof at the broadest part above the main wales, half of which breadth shall be accounted the depth of such vessel, and then deduct from the length three-fifths of the breadth; multiply the remainder by the breadth, and the product by the depth, and divide this last product by 95, for the tonnage. But if it be a single decked vessel, take the length and breadth, as directed above; deduct from the said length three-fifths of the breadth, and take the depth from under the side of the deck plank to the ceiling in the hold, then multiply and divide as aforesaid, and the quotient shall be deemed the

SECTION 2.

The Carpenters' or Sliding Rule.

THIS rule is not only useful in measuring timber, artificers' work, and taking dimensions, but also in ascertaining the contents of such work; it is therefore a rule which all mechanics, having any thing to do with mensuration, ought to possess and understand.

It consists of two equal pieces of box, each one foot in length, connected together by a folding joint; in one of these equal pieces, there is a slider, and four lines marked at the right-hand with the letters A, B, C, D; two of these lines are upon the slider, and the other two upon the rule. Three of these lines, *viz.* A, B, C, are called double lines, because they proceed from 1 to 10 twice over; these three lines are alike both in number and division. They are numbered from the left-hand towards the right with the figures 1, 2, 3, 5, 6, 7, 8, 9, then 1, which stands in the middle; the numbers then proceed, 2, 3, 4, 5, 6, 7, 8, 9, and 10, which stands at the right-hand end of the rule. These numbers have no determinate value of their own, but depend upon the value you set upon the unit at the left-hand of this part of the rule; thus, if you call it 1, the 1 in the middle will be 10, the other figures which follow, will be 20, 30, &c. and the 10 at the right-hand will be 100. If you call the first or left-hand unit, 10, the middle 1 will be 100, and the following figures will be 200, 300, &c. and the 10 at the right-hand end will be 1000. Or if you call the first or left-hand unit 100, the middle 1 will be 1000, the following figures 2000, 3000, &c. and the 10 at the right-hand 10,000. Lastly, as you alter, or number the large divisions, so you must alter the small divisions in the same proportion.

The fourth line D is a single line, proceeding from 4 to 40; it is also called the girt line, from its use in casting up the contents of timber. Upon it are marked W G at 17, 15, and A G at 18, 95, the wine and gauge points, to make it serve the purpose of a gauging rule.

The use of the double lines, A and B, is for working the rule of proportion, and finding the areas of plane figures. On the other part of this side of the rule there is a table to

ascertain the value of a ton, or 50 cubic feet of timber at all prices from 6 pence to 24 pence per foot.

On the other side of the rule are several plain scales divided into 12th parts, marked *inch*, $\frac{3}{4}$, $\frac{1}{2}$, $\frac{1}{4}$, &c. signifying that the inch, $\frac{3}{4}$ inch, &c. are divided into 12 parts. These scales are useful for planning dimensions, that are taken in feet and inches. Again, the edge of the rule is divided into inches, and each of these into eight parts, representing half inches, quarter inches, and half quarters.

In this description the rule is folded; but when it is opened and the slider drawn out, the back part will be found divided like the edge of the rule, so that all together will measure 3 feet or one yard.

USE OF THE CARPENTERS' RULE.

1st. *To multiply numbers together.*

EXAMPLE.

1. Suppose the two numbers 13 and 24.

Set 1 on B to 13 on A; then against 24 on B, stands 312 on A, which is the required product of the two given numbers 13 and 24.

Note. In any operation when a number runs beyond the end of the line, seek it on the other radius, or other part of the line; that is, take the 10th part of it, or the 100th part of it, &c. and increase the product of it proportionally 10 fold, or 100 fold, &c.

2.	Multiply 12	by 16	*Ans.* 192.
3.	35	19	665.
4.	270	54	14580.

2d. *Division of numbers by the Carpenters' Rule.*

EXAMPLE.

1. Required to divide 360 by 12.

Set the divisor 12 on B to the dividend 360 on A; then against 1 on B stands 30, the quotient on A.

2.	Divide 665	by 19.	Quotient 35.
3.	396	27.	14,6.
4.	741	42.	17,6.
5.	7680	24.	320.

Note. In this last example, because 7680 is not contained on A, one-tenth of the number, *viz.* 768 is taken, to make it fall within the compass of the scale—The quotient of this sum is 32, but as the dividend was diminished by a division of 10, so the quotient must be multiplied by the same number, and $32 \times 10 = 320$.

3d. *To square numbers by the Carpenters' Rule.*

EXAMPLE.

1. Required to square the number 25.

Set 1 or 100 on B to the 10 on D; then against every number on D stands its square on the line C. Thus against 25 on D, stands 625, its square on C.

2. Required the square of 30. *Ans.* 900.
3. Required the square of 35. 1225.
4. Required the square of 40. 1600.

Note. If the given number be hundreds, &c. reckon the 1 on D for 100, or 1000, &c. then the corresponding one on C is 10000, or 100000, &c., thus the square of 230 is found to be 52900.

4th. *To find a fourth proportional to three numbers: or to perform the Rule of Three by the Carpenters' Rule.*

EXAMPLE.

1. Required to find a fourth proportional to 12, 28, and 114.

Set the first term 12 on B to the second term 28 on A; then against the third term 114 on B, stands 266 on A, which is the fourth proportional sought.

2. Required the fourth proportional term to the numbers 25 : 75 :: 100. *Ans.* 300.
3. Required the fourth proportional term to the numbers 27 : 20 :: 73. *Ans.* $54\frac{2}{27}$

5th. *To extract the Square Root of any number by the Carpenters' Rule.*

EXAMPLE.

1. Required the square root of 400.

Set 1 upon C to 10 upon D; then against the number 400 on C, stands its root 20 on D.

2. Required the square root of 529. Ans. 23.
3. *What is the* square root of 900? 30.
4. *What is the* square root of 3001 1734

SECTION 3.

Measuring of Boards and Timber.

1st *To find the superficial content of a board or plank.*

RULE.

Multiply the length by the mean breadth. When the board is broader at one end than the other, add the breadth of the two ends together, and take half the sum for the mean breadth.

By the Carpenters' Rule.

Set 12 on B to the breadth in inches on A; then against the length in feet on B, you will find the superficies on A, in feet.

EXAMPLES.

1. How many feet are there in a board that is 13 feet long and 16 inches broad?

Operation.

By duodecimals.				By decimals.	
Ft.	*in.*			13,	
13	0			1,33	
	1	4			
13	0			39	
4	4	0		39	
				13	
17	4	0	*Ans.*	17,29	*Ans.*

By the Carpenters' Rule—

As 12 on B : 16 on A :: 13 on B : $17\frac{1}{3}$ on A. *Ans.*

2. Required the superficies of a board, whose mean breadth is 1 foot 2 inches, and length 12 feet 6 inches?

Ans. 14 feet 7 inches.

3. Required the value of 5 oaken planks, at 3 cents per foot; each of them being $17\frac{1}{2}$ feet long, and their several breadths as follows, viz.; two of $13\frac{1}{2}$ inches in the middle, one of $14\frac{1}{2}$ inches in the middle, and the two remaining ones, each 18 inches at the broader end, and $11\frac{1}{4}$ inches at the narrower? *Ans.* 3 dolls. $9\frac{1}{2}$ cts.

2d. Having the breadth of a board or plank in inches, to find how much in length will make a foot, or any other assigned quantity.

RULE.

Divide 144, or the area to be cut off, by the breadth in inches, and the quotient will be the length in inches.

EXAMPLE.

1. How many inches in length will it require to make one foot, of a board that is 9 inches broad?

Operation. $144 \div 9 = 16$ inches, the length required.

2. How many inches in length, of a board that is 23 inches wide, will make 1 foot?

Ans. 6,26+inches.

3. From a mahogany plank 26 inches broad, a yard and a half (or 13 feet 6 inches) is required to be cut off; what distance from the end must the line be struck?

Ans. 74,7692 inches, or 6,23 feet.

3d. To find the solid content of squared or four sided timber.

RULE.

Multiply the mean breadth by the mean thickness, and that product by the length; the last product will give the solid content.

Note. 1. If the tree taper regularly from one end to the other, take the mean breadth and thickness in the middle; or take half the sum of the dimensions at the two ends, for the mean dimension.

2. If the piece does not taper regularly, take several different dimensions, add them all together, and divide their sum by their number, for the mean dimension.

3. The quarter girt is an arithmetical mean, proportional between the mean breadth and thickness, that is the square root of the product.

EXAMPLES.

1. If a piece of timber be 2 feet 9 inches deep, and 1 foot 7 inches broad, and the length 16 feet 9 inches, (or which

is the same thing,* if the quarter girt be 26 inches, and the length 16 feet 9 inches,) how many solid feet are contained therein?

Operation.

26 inches quarter girt	16,75 = 16 feet 9 inches, the length.
26	676
156	10050
52	11725
676 square	10050
	144) 11323,00 (78,63 + feet. *Ans.*
	1008
	1243
	1152
	910
	864
	460
	432
	28 rem.

By the Carpenter's Rule—

As 12 on D : 16¾ on C :: 26 on D : 78½ on C. *Ans.*

2. The quarter girt of a piece of squared timber is 15 inches, and the length 18 feet; required the solidity?

Ans. 28⅛ feet.

3. If a piece of squared timber be 25 inches square at the greater end, and 9 inches square at the less, and the length be 20 feet; what is the solid content?

Ans. 40,13 feet.

4. Suppose a piece of squared timber to measure 32 by 20 inches at the greater end, and 10 by 6 inches at the less, and the length 18 feet; how many feet of timber are contained therein? *Ans.* 34,12 + feet.

* This is making use of an arithmetical mean, instead of a geometrical one, which is not exactly true; it is, however, sufficiently exact for common purposes, when the timber is nearly square; the error increases, the more the breadth and depth differ from each other.—[illegible]ter exactness is required, multiply the breadth by the [illegible]iddle, and that product by the length, for the true ca[illegible]

4th. To find the solid content of round timber.

RULE.

Multiply the square of the quarter girt, or of ¼ of the mean circumference, by the length, for the content.

Note. 1. To find the quarter girt of round timber, measure round the middle with a line, one-fourth part of this is reckoned the quarter girt.

2. When the tree is tapering, take either the mean dimension, as in squared timber, or girt it at both ends, and take half the sum. If the tree is very irregular, divide it into several lengths, add all the girts together, and divide the amount by the sum of them, for the mean girt.

3. The buyer is allowed to take the girt anywhere between the greater end and the middle, if it taper; an allowance must also be made for bark; *one-tenth* for oak, but less for ash, beech, &c.

EXAMPLES.

1. A piece of round timber being 9 feet 6 inches long, and its mean quarter girt 42 inches, what is the content?

Operation.

Decimals.	Duodecimals.
3,5=42 inches, quarter girt.	*Ft. in.*
3,5	3 6=42 inches.
	3 6
175	
105	1 9 0
	10 6
12,25	
9,5 length	12 3 0
	9 6
6125	
11025	6 1 6
	110 3
116,375 content.	
	116 4 6

By the Carpenters' Rule—

As 9,5 on C : 10 on D :: 3,5 on D : 116¼ on C
Or, 9,5 on C : 12 on D :: 42 on D : 116¼ on C } *Ans.*

2. The length of a tree is 24 feet, its girt at the thicker end is 14 feet, and at the smaller end 2 feet; what is its content? Ans. 96 feet

3. If a piece of round timber 18 feet long, measure 96 inches in circumference, or the quarter girt 24 inches; how many feet of timber does it contain?

Ans. 72 feet.

4. If a piece of round timber measure 11 feet 4 inches at the larger end, 2 feet 8 inches at the less, and its length 21 feet, how many feet of timber are contained therein?

Ans. 64,31 feet.

5. Required the amount of three pieces of round timber measuring as follows, *viz.*

The first	24	feet long and mean girt		8 feet,
The second	14½	do.	do.	3,15
The third	17¼	do.	do.	6,28

Ans. 147 5+ feet.

SECTION 4.

OF CARPENTERS' AND JOINERS' WORK.

To this branch belongs all the wood-work of a house, such as framing, flooring, partitioning, roofing, &c.

Carpenters usually measure their work by the square, (consisting of 100 superficial feet) the yard or foot; but enriched mouldings, cornices, &c. are estimated by running or lineal measure, and some things are rated by the piece.

In measuring of Carpenter's work, the string is made to ply close to every part of the work over which it passes.

Partitions are measured from wall to wall for one dimension, and from floor to floor, as far as they extend, for the other.

In framing, no deductions are made for door-ways, fire-places, or other vacancies, on account of the additional trouble of framing arising from them.

For stair-cases, take the breadth of all the steps, by *making* a line ply close over them, from the top to the bottom, and multiply the length of this line, by the length of a step, for the whole area. By the length of a step is

meant the length of the front, and the returns at the two ends; and by the breadth, is to be understood the girt of its two outer surfaces, or the tread and rise.

The rail of a stair-case is taken at so much per foot in length, according to the diameter of the well-hole; architrave string boards, by the foot superficial; brackets and strings at so much per piece, according to the workmanship.

Wainscoting is measured by the yard square, consisting of 9 feet.

Door cases, frame doors, modillion cornices, eaves, frontispieces, &c. are generally measured by the foot superficial.

Joists are measured by multiplying their breadth by their depth, and that product by their length. They receive various names, according to the place in which they are laid to form a floor; such as trimming joists, girders, binding joists, bridging joists, ceiling joists, &c.

In boarded flooring, the dimensions must be taken to the extreme parts; and the number of squares of 100 feet, are to be calculated from these dimensions. Deductions must be made for chimneys, stair-cases, &c.

In roofing, take the whole length of the timber, for the length of the framing, and gird over the ridge from wall to wall with a line, for the breadth. This length and breadth multiplied together give the content.

In measuring of roofing for work and materials, all holes for chimney-shafts, sky-lights, &c. are included in the measurement, on account of their trouble and waste of materials; but for workmanship alone, they are generally deducted.

It is a common rule among carpenters, that the flat of any house, and half the flat thereof taken within the walls, is equal to the measure of the roof of the same house; this is, however, only when the roof is the true pitch—where the length of the rafters are $\frac{3}{4}$ of the breadth of the building. The pitch of roofs varies according to the materials with which they are covered, and fancy of the builder.

Weather-boarding, like flooring, is measured by the *square, and sometimes* by the yard.

EXAMPLES.

1. If a floor be 57 feet 3 inches long, and 28 feet 6 inches broad, how many squares of flooring does it contain?

Operation.

By decimals.

Ft.	*in.*	
57	3	=57,25
28	6	= 28,5
		28625
		45800
		11450
		100)1631,625
		16,31,625

By duodecimals.

Ft.	*in.*	
57	3	
28	6	
		456
		114
		28 7 6
		7 0 0
		100)1631 7 6
		16,31 7 6

Ans. 16 squares 31 feet 7 in. 6′

2. Let a floor be 53 feet 6 inches long, and 47 feet 9 inches broad, how many squares does it contain?

Ans. 25 squares 54 feet.

3. A floor being 36 feet 3 inches long, and 16 feet 6 inches broad, what will it cost at 4 dollars and 50 cents per square? *Ans.* 26 dolls. 91 cents.

4. A room is 35 feet long and 30 feet wide; there is in it a fire-place which measures 6 feet by 4 feet 6 inches, and a well-hole for the stairs measures 10 feet 6 inches by 8 feet; what will the flooring come to at 3 dollars and 75 cents per square? *Ans.* 35 dolls. 21+cts.

5. How many squares are contained in a partition that is 62 feet 6 inches long, and 12 feet 3 inches high?

Ans. 10 squares and 10+feet.

6. If a partition between rooms be in length 91 feet 9 inches, and its height 11 feet 3 inches; how many squares are contained in it, and how much does it come to at 4 dollars 50 cents per square?

Ans. 10 squares 32 feet, and costs 46 dolls. 44 cts.

7. If a house within the walls be 44 feet 6 inches long, and 18 feet 3 inches broad; how many squares of roofing will it contain, allowing the roof to be the true pitch?

Operation.

By decimals.

Ft.	*Ft.*	*in.*	
18,25=	18	3	the breadth.
44,5=	44	6	the length.
9125			
7300			
7300			
Flat 812,125			
Half 406,062			
÷100) 1218,187			
Sum 12,18+			

By duodecimals.

18	3	
44	6	
72		
72		
792		
11	0	0″
9	1	6
Flat 812	1	6
Half 406+		
÷100) 1218	1	6
Sum 12,18		

Ans. 12 sq. 18 ft.

8. What cost the roofing of a house at 1 dollar and 40 cents per square; the length within the walls being 52 feet 8 inches, and the breadth 30 feet 6 inches; the roof being of a true pitch?

Ans. 33 dollars 73 cents.

9. Suppose a house measures, within the walls, 40 feet 6 inches in length, and 20 feet 6 inches in breadth, and the roof being a true pitch; how many squares of roofing does it contain, and how much will it cost at 2 dollars 25 cents per square?

Ans. 12,45375 squares, and costs 28 dolls. 2+cts.

Note. All timbers in a roof are measured in the same manner as in floors, except king-posts, which are measured by taking their breadth and depth at the widest place, and multiplying these together, and the product by the length.

10. If a room or wainscot, being girt downwards over

the mouldings, be 15 feet 9 inches high, and 126 feet 3 inches in compass; how many yards does that room contain?

Operation.

By decimals.	By duodecimals.
126,25	*Ft.* *in.*
15,75	126 3
	15 9
63125	630
88375	126
63125	1890
12625	63 1 6
	31 6 9
9)1988,4375	3 9 0
	9)1988 5 3
Sum 220,8	Sum 220 8

Ans. 220 yds. 8 feet.

11. If a room of wainscot be 16 feet 3 inches high, and the compass of the room 137 feet 6 inches; how many yards are contained in it? *Ans.* 248 yards 2+ feet.

12. If the window-shutters about a room be 69 feet 9 inches broad, and 6 feet 3 inches high; how many yards are contained therein, at work and half?

Ans. 72,656 yards.

13. What will the wainscoting of a room come to at 80 cents per square yard, supposing the height of the room, including the cornice and moulding, be 12 feet 6 inches, and the compass 83 feet 8 inches; three window-shutters, each 7 feet 8 inches by two feet 6 inches, and the door 7 feet by 3 feet 6 inches; the shutters and door being worked on both sides, are reckoned work and half?

Ans. 96 dollars 60¾ cents.

SECTION 5.

OF BRICKLAYERS' WORK.

Brick Work is measured and estimated in various ways. In *some places* walls are measured by the rod square of 16½ *feet; so that* one rod in length, and one in breadth

contain 272,25 square feet; in other places the custom is to allow 18 feet to the rod, that is, 324 square feet.

In other places they measure by the rod of 21 feet long, and 3 feet high, that is, 62 square feet. Again, in other places they account 16½ feet long and 1 foot high, that is, 16½ square feet, a rod or perch; or again, by the yard of 9 square feet; and oftentimes the work is estimated at so much per thousand bricks.

When brick work is measured by the rod, or perch, it must be estimated at the rate of a brick and a half thick; so that if a wall be more or less than this standard thickness, it must be reduced to it by the following

RULE.

Multiply the superficial content of the wall by the number of half bricks in the thickness, and divide the product by 3 for the superficial feet in standard thickness.

EXAMPLES.

If a wall be 72 feet 6 inches long, and 19 feet 3 inches high, and 5 bricks and a half thick; how many rods of brick work are contained therein, when reduced to the standard thickness?

Operation.

By decimals.

19,25 = the height
72,5 = the length.
9625
3850
13475
1395,625
11 = the thickness.
÷3) 15351,875
÷272,25) 5117,291 (18 rods.
239479
68,06) 216,79 (3 quarters.
12,61

By duodecimals.

Ft.	in.	
72	6	
19	3	
648		
72		
1368		
18	1	6′
9	6	
1395	7	6
		11

÷3) 15351 10 6
272) 5117 (18 rods.
2397
68) 221 (3 quarters
17 feet.

Note. Observe that 68,06 is the one-fourth part of 272,25, and 68 is only the one-fourth part of 272. As the number 272½ is an inconvenient number to divide by, the ½ is usually omitted, and the content in feet divided only by 272; the difference being too trifling to be considered in practice.

To find fixed divisors for bringing the answer into feet or rods of a standard thickness, without multiplying the superficies by the number of half bricks, &c.

RULE.

Divide three, the number of half bricks in 1½, by the number of half bricks in the thickness, the quotient will be a divisor, which will give the answer in feet. Or if a divisor is sought for, that will bring the answer in rods at once, multiply 272 by the divisor found for feet, and the product will be a divisor for rods; as in the following

TABLE.

1 *The thickness of the wall.*	2 *Divisors for the answer in feet.*	3 *Divisors for the answer in rods.*
1 brick	1,	408
1½ brick	1	472
2 bricks	,75	204
2½ bricks	,6	163,2
3 bricks	,5	136
3½ bricks	,4285	116,6
4 bricks	,375	102
4½ bricks	,3333	90,6
5 bricks	,3	81,6
5½ bricks	,2727	74,18

Application of the above Table.

Multiply the length of the given wall by the breadth, observe the number of half bricks it is in thickness; and opposite thereto will be found in the second column the divisor to reduce it to feet. and in the third column the divisor for rods. Thus in the above example 72,5 × by 19,25= 1395,625.

And 1395,625÷2727=5117+ the number of feet in standard measure.

And 1395,625÷74,18=18,8+ the number of rods.

Or, by the Carpenters' Rule—

As the tabular divisor, against the thickness of the wall : is to the length of the wall :: so is the breadth : to the content.

As 74,18 on B : 72,5 on A :: 19,25 on B : 18¾ on A. *Ans.*

To find the dimensions of a building, measure half around on the outside, and half on the inside, for the whole length of the wall; this length being multiplied by the height gives the superficies. All the vacuities, such as doors, windows, window-backs, &c. must be deducted, for materials; but for workmanship alone no deductions are to be made, and the measurement is usually taken altogether on the outside. This is done in consideration of the trouble of the returns or angles. There are also some other allowances, such as double measure for feathered gable ends, &c.

2. How many yards and rods of standard thickness are contained in a brick wall, whose length is 57 feet 3 inches, and height 24 feet 6 inches; the wall being 2½ bricks thick?
Ans. 259,74 yards, or 8,58+rods.

3. If a wall be 245 feet 9 inches long, 16 feet 6 inches high, and 2½ bricks thick; how many rods of brick work are contained therein, when reduced to standard thickness?
Ans. 24 rods 3 quarters 24 feet.

4. A triangle gable end is raised to the height of 15 feet above the wall of a house, whose width is 45 feet and the thickness of the wall 2½ bricks; required the content in rods at standard thickness? *Ans.* 2 rods 18 feet.

Chimneys by some are measured as if they were solid, deducting only the vacuity from the hearth to the mantle, on account of their trouble.

But by others, they are girt or measured round for their breadth, and the height of their story, taking the depth of the jambs for their thickness. And in this case no deduction is made for the vacuity from the floor to the mantle-tree, *because of the* gathering of the breast and wings, to make room for the hearth in the next story.

If the chimney back be a party wall, and the wall be measured by itself, then the depth of the two jambs and length of the breast is to be taken for the length, and the height of the story for the breadth, at the same thickness with the jambs.

Those parts of the chimney-shaft which appear above the roof are to be girt with a line round about the least place of them for the length, and take the height for the breadth; and if they are 4 inches thick, they are to be accounted as one brick work, and if they are 9 inches thick, they are to be taken for 1½ brick work, on account of the trouble of plastering and scaffolding.

It is customary in most places to allow double measure for chimneys.

SECTION 6.

OF MASONS' WORK.

Masons' work is measured sometimes by the foot solid, sometimes by the foot superficial, and sometimes by the foot in length. It is also measured by the yard, and mostly by the rod or perch, which is 16½ feet in length, 18 inches in breadth, and 12 inches in depth.

Walls are measured by the perch; columns, blocks of stone or marble, &c. by the cubic foot; and pavements, slabs, chimney-pieces, &c. by the superficial or square foot.

Cubic, or solid measure, is always used for materials, but square measure generally for workmanship.

In solid measure, the true length, breadth and thickness are taken and multiplied into each other for the content.

In superficial measure, the length and breadth of every projection, which is seen without the general face of the building, is taken for the content.

EXAMPLES.

1. If a wall be 97 feet 5 inches long, 18 feet 3 inches high, and 2 feet 3 inches thick, how many solid feet, and perches, are contained therein?

Operation.

By decimals.		By duodecimals.				
97,417	length	*Ft.*	*in.*			
18,25	breadth	97	5			
		18	3			
487085						
194834		776				
779336		97				
97417		24	4	3″		
		6	0	0		
1777,86025	superficies	1	6	0		
2,25	thickness					
		1777	10	3		
888930125				2	3	
355572050						
355572050		3555	8	6		
		444	5	6	9‴	
4000,1855625	solidity.					
	in cubic ft.	4000	2	0	9	*Ans.*

4000÷24,75=161,616+feet. *Ans.*

2 How many solid feet and perches are contained in a wall 53 feet 6 inches long, 12 feet 3 inches high, and 2 feet thick? *Ans.* 1310,75 feet, and 52,9595 rods.

3. If a wall be 107 feet 9 inches long, and 20 feet 6 inches high, how many superficial feet are contained therein?
Ans. 2208 feet 10 inches.

4. If a wall be 112 feet 3 inches long, and 16 feet 6 inches high, how many superficial rods, each 63 square feet, are contained therein? *Ans.* 29 rods 25 feet.

5. What is a marble slab worth, whose length is 5 feet 7 inches, and breadth 1 foot 10 inches, at 80 cents per foot *superficial?* Ans. 8 dolls. 19 cts.

SECTION 7.

OF PLASTERERS' WORK.

PLASTERERS' WORK is principally of two kinds, *viz.*—*first*, plastering upon laths, called ceiling; and *second*, plastering upon walls, or partitions made of framed timber, called rendering; which are measured separately.

Plasterers' work is usually measured by the yard square, consisting of 9 square feet; sometimes it is measured by the square foot, and sometimes by the square of 100 feet.

Enriched mouldings, cornices, &c. are rated by running, or lineal measure. In arches, the girt round them multiplied by the length, is taken for the superficies.

Deductions are to be made for doors, chimneys, windows, and other large vacuities. But when the windows, or other openings, are small, they are seldom deducted, as the plastered returns at the top and sides are allowed to compensate for the vacuity.

Whitewashing and coloring are measured in the same manner as plastering.

EXAMPLES.

1. If a ceiling be 59 feet 9 inches long, and 24 feet 6 inches broad, how many superficial yards of 9 square feet does it contain?

Operation.

By decimals.	By duodecimals.
Ft. in.	*Ft. in.*
59 9=59,75 feet	59 9
24 6= 24,5 do.	24 6
29875	236
23900	118
11950	29 10 6″
÷9) 1463,875 feet	18 0 0
,65+ yards	÷9) 1463 10 6
	162 5 10 6 Ans.

2. If the plastered partitions between rooms be 141 feet 6 inches about, and 11 feet 3 inches high, how many yards do they contain? *Ans.* 176,87 yards.

3. What will the plastering of a ceiling come to at 15 cents per yard, allowing it to be 22 feet 7 inches long, and 13 feet 11 inches broad? *Ans.* 5 dolls. 20 cts.

4. The length of a room being 20 feet, its breadth 14 feet 6 inches, and height 10 feet 4 inches; how many yards of plastering does it contain, deducting a fire-place of 4 feet by 4 feet 4 inches, and two windows, each 6 feet by 3 feet 2 inches? *Ans.* $73\frac{2}{27}$ yards.

5. The length of a room is 14 feet 5 inches, breadth 13 feet 2 inches, and height 9 feet 3 inches, to the under side of the cornice, which projects 5 inches from the wall, on the upper part next the ceiling; required the quantity of rendering and plastering; there being no deductions but for one door, the size whereof is 7 by 4 feet?

Ans. 53 yds. 5 ft. of rendering, and 18 yds. 5 ft. ceiling.

6. The circular vaulted roof of a church measures 105 feet 6 inches in the arch, and 275 feet 5 inches in length; what will the plastering come to at 12 cents per yard?

Ans. 387 dolls. 42 cts.

7. What will the whitewashing of a room come to at 2 cents per yard, allowing it to be 30 feet 6 inches long, 24 feet 9 inches broad, and 10 feet high; no deductions being made for vacuities? *Ans.* 4 dolls. $13\frac{1}{2}$ cts.

SECTION 8.

OF PAVERS' WORK.

PAVERS WORK is measured by the square yard, consisting of 9 square feet. The superficies is found by multiplying the length by the breadth.

EXAMPLES.

1. What cost the paving of a street 225 feet 6 inches long, and 60 feet 6 inches wide, at 30 cents per square yard?

By decimals.

Ft. *in.*
225 6=225,5 feet
60 6= 60,5 do.

11275
13530

÷9)13642,75 superficial ft.

1515,86 yards.
30

Ans. 454,7580

By duodecimals.

Ft.	*in.*	
225	6	
60	6	
13500		
112	9	
30		
9)13642	9	
1515	7	9
30		
454,50		

26=the price of 7 ft. 9 in.

454,76

Ans. 454 dolls. 76 cts.

2. What will the paving of a foot-path come to at 28 cents per yard, the length being 35 feet 4 inches, and the breadth 8 feet 3 inches? *Ans.* 9 dolls. 33 cts.

3. What cost the paving of a court-yard at 38 cents per yard, the length being 27 feet 10 inches, and the breadth 14 feet 9 inches? *Ans.* 17 dolls. 33¼ cts.

4. What will be the expense of paving a rectangular yard, whose length is 63 feet, and breadth 45 feet, in which there is laid a foot-path 5 feet 3 inches broad, running the whole length, with broad stones, at 36 cents a yard; the rest being paved with pebbles, at 30 cents a yard?

Ans. 96 dolls. 70½ cts.

SECTION 9.

OF PAINTERS' WORK.

Work is computed in square yards of 9 feet measured where the color lies; and the measure pressed close into all the mouldings, corners, &c. it passes.

Windows, casements, &c. are estimated at so much a piece; and it is usual to allow double measure for carved mouldings, &c.

The value of painting is rated by the number of coats; or whether once, twice, or thrice colored over, and the different qualities and costliness of the colors.

EXAMPLES.

1. How many yards of painting will a room contain which (being girt over the mouldings) is 16 feet 6 inches, and the compass of the room 97 feet 6 inches?

Operation.

By decimals.	By duodecimals.
Ft. *in.*	97 6
97 6=97,5	16 6
16 6=16,5	
	583
4875	97
5850	48 9
975	8 0
÷9) 1608,75 feet	÷9) 1608 9
Yards 178,6,75	178,6,9

Ans. 178¾ yards.

2. A gentleman had a room painted at 8½ cents per yard, the measure whereof is as follows, *viz.* the height 11 feet 7 inches, the compass 74 feet 10 inches, the door 7 feet 6 inches by 3 feet 9 inches; five window shutters, each 6 feet 8 inches by 3 feet 4 inches; the breaks in the windows 14 inches deep, and 8 feet high; the opening for the chimney 6 feet 9 inches by 5 feet, to be deducted, the shutters and doors are painted on both sides; what will the whole come to?

Ans. 10 dolls. 43 cts.

3. How many yards of painting are there in a room, the length whereof is 20 feet, its breadth 14 feet 6 inches, and height 10 feet 4 inches; deducting a fire-place of 4 f 4 feet 4 inches, and two windows, each 6 feet by 4 *inches?* Ans. 78¾

4. What cost the painting of a room at 6 cents per yard, its length being 24 feet 6 inches, its breadth 16 feet 3 inches, and height 12 feet 9 inches; also the door is 7 feet by 3 feet 6 inches, and the window shutters of two windows, each 7 feet 9 inches by 3 feet 6 inches, but the breaks of the windows themselves, are 8 feet 6 inches high, and 1 foot 3 inches deep; deducting a fire-place of 5 feet by 5 feet 6 inches. *Ans.* 7 dolls. 66 cts. 9½ m.

SECTION 10.

OF GLAZIERS' WORK.

GLAZIERS compute their work in square feet; and the dimensions are taken either in feet, inches, and seconds, &c. or in feet, tenths, hundredths.

Windows are sometimes measured by taking the dimensions of one pane, and multiplying its superficies by the number of panes. But more generally they measure the length and breadth of the window over all the panes, and their frames for the length and breadth of the glazing. And oftentimes the work is estimated at so much per pane according to the size.

Circular, or oval windows, as fan-lights, &c. are measured as if they were square, taking for their dimensions the greatest length and breadth, as a compensation for the waste of glass, and labor in cutting it to the proper forms.

EXAMPLES.

1. How many square feet are contained in a window, which is 4 feet 3 inches long, and 2 feet 9 inches broad?

By decimals.

Ft.	*in.*	
4	3=4,25	the length
2	9=2,75	the breadth
	2125	
	2975	
	850	
		feet.

By duodecimals.

Ft.	*in.*		
4	3		
	2	9	
8	6		
3	2.	3	
11	8	3	*Ans.*

2. If a window be 7 feet 3 inches high, and 3 feet 5 inches broad, how many square feet of glazing are contained therein? *Ans.* 24 feet 9 inches.

3. There is a house with three tiers of windows, 7 in a tier; the height of the first tier is 6 feet 11 inches, of the second, 5 feet 4 inches, and of the third, 4 feet 3 inches; the breadth of each window is 3 feet 6 inches: what will the glazing come to at 14½ cents per foot?

Ans. 58 dolls. 61 cts.

4. What will the glazing of a triangular sky-light come to at 10 cents per foot, the base being 12 feet 6 inches long, and the perpendicular height 16 feet 9 inches?

Ans. 10 dolls. 46¾ cts.

5. What is the area of an elliptical fan-light of 14 feet 6 inches in length, and 4 feet 9 inches in breadth?

Ans. 68 feet 10 inches.

6. There is a house with three tiers of windows, and 9 in each tier; the height of the first tier is 7 feet 10 inches, of the second, 6 feet 8 inches, of the third, 5 feet 4 inches, and the common breadth 3 feet 11 inches; what will the glazing come to at 14 cents per foot?

Ans. 97 dolls. 87¾ cts.

SECTION 11

MEASUREMENT OF GROUND.

1*st. To find the content of a square piece of ground.*

RULE.

MULTIPLY the base in perches, yards or feet, as the case may be, by the perpendicular, and the product will be the answer required.

Note. 1. Any area, or content in perches, being divided by 160, will give the content in acres; the remaining perches, if more than 40, being divided by 40, will give the roods, or quarter acres, and the last remainder, if any, will be perches.

2. Ground is generally measured by chains, of two poles or rods in length; the two pole chain measures 33 feet. Chains of 4 poles are *sometimes used*, and sometimes chains or poles of one rod *only*.

EXAMPLE.

1. In a square field, A, B, C, D, each side of which measures 40 rods, or poles, how many acres?

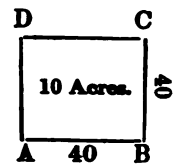
Operation.

40
40

4,0) 160,0

4) 40

10 acres. *Ans.*



2. In a square field, each side of which measures 35 two pole chains, how many acres?

Ans. 30 acres 2 roods 20 perches.

3. A piece of square ground measures $16\frac{1}{2}$ perches on each side; what is the content in acres?

Ans. 1 acre 2 roods $32\frac{1}{4}$ perches.

2d. *To find the content of an oblong square piece of ground, called a parallelogram.*

RULE.

Multiply the length by the breadth, and the product will be the answer.

EXAMPLE.

1. There is an oblong square piece of ground, A, B, C, D, the longest sides of which measure 64 perches, and the shortest sides, or ends, measure 40; how many acres does it contain?

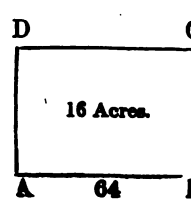
Operation.

64=the length
40=the breadth

4,0) 256,0 perches

4) 64

acres. *Ans.*



2. In a piece of ground lying in the form of an oblong square, the length measures 120 perches, and the breadth 84; what is its content in acres? *Ans.* 63 acres.

3. A lot of ground lying in the form of an oblong square, measures 240 feet in length, and 120 in breadth; what is its content in acres?

Ans. 0 acres 2 quarters 25 perches 213¾ feet.

4. There is an oblong piece of ground, whose length is 14 two pole chains 25 links, and breadth 8 chains 37 links; how many acres does it contain?

Ch.	*L.*	*Perches.*
8	37	=17,48 breadth
14	25	= 29 length

15732
3496

4,0)50,6,92

4)12,26

3 0 26,92

Ans. 3 acres 0 quarters 27 perches nearly.

Note. The *English statute perch* is 5½ yards, the two pole chain is 11 yards, or 33 feet, and is divided into 50 links; the four pole chain is 22 yards, or 66 feet, and contains 100 links; hence the length of a link in a statute chain is 7,92 inches, and 25 links make 1 rod. And consequently, if the links be multiplied by 4, carrying 1 to the chains for every 25 links, and the chains by 2, the product will be perches, and decimals of a perch.

5. An oblong piece of ground measures 17 two pole chains and 21 links in length, and 15 chains 38 links in breadth; how many acres are contained therein?

Ch.	*L.*		*Ch.*	*L.*
17	21		15	38
2	4		2	4
34	84	perches, the length.	31	52 p. the breadth.

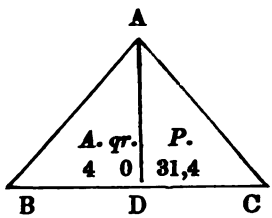
Then 34,84 × 31,52=1098,1568 perches=6 acres 3 qr. 18,15+perches.

3*d*. *To find the content of a triangular piece of ground.*

RULE.

Multiply the base by half the perpendicular, or the perpendicular by half the base, or take half the product of the *base into the* perpendicular.

EXAMPLE.



1. Let A, B, C, be a triangular piece of ground, the longest side or base B, C, is 24 chains 38 links, and perpendicular, A D, 13 chains 28 links; how many acres does it contain?

Operation.

Ch.	*L.*
24	38=49,52 perches
13	28=27,12

9904
4952
34664
9904

1342,9824

Half the sum is 4,0)67,1,4912 perches

4)16,31

4 0 31,4

Ans. 4 acres 0 roods 31,4 perches.

2. In a triangular piece of ground, the base or longest side measures 75 perches, and the perpendicular 50; how many acres does it contain?

Ans. 11 acres 2 qrs. 35 perches.

3. How much will a triangular piece of ground come to at 45 dollars per acre, the longest side or base of which measures 120 perches, and the perpendicular 84 perches.

Ans. 1417 dolls. 50 cents.

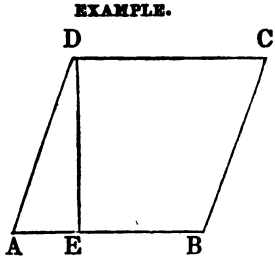
4. How many superficial yards are contained in a triangular piece of ground, the base of which measures 140 feet perpendicular 70 feet? Ans 544 yards 4 feet.

4th. To find the content of a piece of ground, in the form of an oblique parallelogram.

RULE.

Multiply the base into the perpendicular height for the content.

EXAMPLE.



. Let A, B, C, D, be a piece of ground in the form of an oblique parallelogram, the base of which, A, B, measures 44 perches, and the perpendicular, D, E, 40 perches; how many acres does it contain?

44 length
40 breadth

4,0) 176,0 perches

4) 44

11 acres. *Ans.* 11 acres.

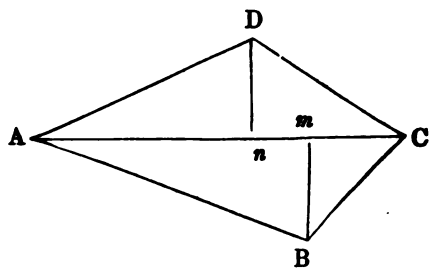
2. A piece of ground lying in the form of an oblique parallelogram, is found to measure 80 perches along its base, and its perpendicular height 24 perches; how many acres does it contain? *Ans.* 12 acres.

5th. To find the content of a piece of ground bounded by four sides, none of which are parallel or equal.

RULE.

Find the length of a diagonal line between the two most distant corners, and multiply this line by the sum of the two perpendiculars falling from the other corners to that diagonal line, and half the product will be the area.

EXAMPLE.



1. Let A, B, C, D, be a field with four irregular and unequal sides, the diagonal line of which, A, C, measures 80 perches, the perpendicular, B, *m*, measures 25 perches, and the other perpendicular, D, *n*, 35 perches; how many acres does it contain?

80 the length of the diagonal line.
25+35=60 the sum of the two perpendiculars.

2)4800

4,0)240,0 perches

4) 60

15 acres. *Ans.*

2. In a field of four unequal sides, the diagonal line between the two most distant corners measures 120 rods, and the perpendiculars measure, the one 48, and the other 24 rods; required the number of acres it contains?

Ans. 27 acres.

6th. To find the area of a piece of ground lying in a circle, or ellipsis.

RULE.

Multiply the square of the circle's diameter, or the product of the longest and shortest diameters of the ellipsis, by the decimal number ,7854, the product will give the area.

Note. In any circle, the *Diameter* multiplied by 3,14159 produces the circumference; *Circumference* divided quotes the diameter.

EXAMPLE.

1. How many acres are contained in a circular piece of ground, whose diameter measures 320 perches, or 1 mile?

320 × 320 = 102400
,7854

409600
512000
819200
716800

4,0) 80424,9600 perches

4) 2010,24,9

502 2 24,9

Ans. 502 acres 2 qr. 24,9 perches.

2. A gentleman has an elliptical yard in front of his house, the longest diameter of which measures 30 perches, and the shortest 20; how much ground is contained therein?

Ans. 2 acres 3 qr. 31,2 perches.

3. How many square yards are contained in a circular piece of ground, the diameter of which measures 160 feet?

Ans. 2234+yards.

From the foregoing simple methods of finding the contents of ground lying in different forms, it will readily be seen, that the content of fields and small pieces of land, lying in any shape whatever, and bounded by any number of sides, may be calculated, without having recourse to the more expensive and troublesome practice of employing a regular surveyor. No other apparatus than a common rod-pole, or line of a known length, is requisite. Pieces of land having more than four boundary lines, may be easily divided into squares, parallelograms, triangles, &c. and each calculated separately by some of the foregoing rules, and then the whole amount added into one sum for the content. It is of great importance to every practical farmer to know the size of the different fields which he cultivates. Besides the satisfaction thereof, this knowledge is necessary to enable him to regulate the quantity of seed which he should sow, as well the price for clearing, plowing, planting, reaping, &c.

SECTION 12.

OF GAUGING.

GAUGING is taking the dimensions of a cask in inches, to find its content in gallons.

RULE.

1. Find the mean diameter, between the head and bung diameters, by adding two-thirds of the difference between them to the head diameter. If the staves be but little curving from the head to the bung, add only six-tenths of this difference.

2. Square the mean diameter, so found, and multiply the product by the length of the cask in inches, for the content thereof in cubic inches.

3. Divide the cubic inches, so found, by 294, for wine or spirits, and by 359 for ale; the quotient will be the answer in gallons.

EXAMPLE.

1. How many gallons of wine will a cask contain whose bung diameter is 31 inches, head diameter 25 inches, and whose length is 3 feet, or 36 inches?

Operation.

31 bung diam.	25	head diameter
25 head diam.	$\frac{2}{3}$ of 6 = 4	two-thirds difference
6 difference.	29	
	29	
	261	
	58	
	841	square of the mean diam.
	36	the length
	5046	
	2523	
	30276	cubic inches.

Then 30276 ÷ 294 = 102$\frac{138}{147}$ gals. Or, 102 gals. 3 qt. 1$\frac{11}{49}$ pt.

2. The diameter of a barrel at the bung measures 24 inches, and at the head 18 inches, and its length is 24 inches; what is its content in wine measure?

Ans. $39\frac{21}{49}$ gals.

3. How many gallons of spirits will a cask contain, whose bung diameter is 36 inches, head diameter 28 inches, and whose length is 3 feet 4 inches?

Ans. $151\frac{327}{1323}$ gals.

4. What is the content, in ale measure, of a barrel whose bung diameter measures 18 inches, head diameter 15 inches, and whose length is 2 feet 5 inches?

Ans. $23\frac{124}{350}$ gals.

5. Bought a barrel of ale of the following dimensions, *viz.* bung diameter 22 inches, head diameter 18 inches, and length 3 feet; how many gallons does it contain?

Ans. $42\frac{218}{350}$ gals.

Of the Gauging or Diagonal Rod.

The diagonal rod is a square rule, having four faces, being commonly four feet long, and folding together by joints. This instrument is used for gauging, or measuring casks, and computing their contents, and that from one dimension only, namely, the diagonal of the cask; that is, from the middle of the bung-hole, to the meeting of the head of the cask, with the stave opposite to the bung; being the longest line that can be drawn within the cask from the middle of the bung-hole. And accordingly one face of the rule is a scale of inches, for measuring this diagonal, to which are placed the areas in ale gallons, of circles to the corresponding diameters, in like manner as the lines on the under sides of the three slides, in the sliding rule. On the opposite face are two scales of ale and wine gallons, expressing the contents of casks having the corresponding diagonals. And these are the lines which chiefly form the difference between this instrument and the sliding rule.

EXAMPLE.

The rod being applied within the cask at the bung,

the diagonal was found to be 34,4 inches; required the content in gallons.

Now, to 34,4 inches, will be found corresponding on the rod, 90¾ ale gallons, and 111 wine gallons, the content required.

Note. In taking the length of a cask to find the cubic inches, an allowance must be made for the thickness of both the heads of 1 inch, of 1½ inch, or 2 inches, according to the size of the cask; and the head diameter must always be taken close to the chime. The contents exhibited by the rod, answer only to casks of the common form.

SECTION 13.

OF MECHANICAL POWERS.

1st. OF THE LEVER.

To find what weight may be raised or balanced by any given power.

RULE.

As the distance between the body to be raised, and fulcrum, or prop,

Is to the distance between the prop, and the point where the power is applied,

So is the power to the weight which it will raise.

EXAMPLE.

1. If a man weighing 150 lb. rest on the end of a lever 12 feet long; what weight will he balance on the other end, supposing the prop 1½ foot from the weight?

Operation.

12=the length of the lever
1,5=distance of the weight from the prop

10,5=the distance from the prop to the man.

as 1,5 : 10,5 :: 150 : 1050. ***Ans.***

of a pair of steelyards weighing 5 lb. is removed back from the fulcrum; what weight will

it balance, suspended at 1 inch distance on the opposite side?
Ans 100 lb.

2d. OF THE WHEEL AND AXLE.

To find what power must be applied at the wheel, to raise a given weight suspended to the axle; or what weight at the axle will be raised by a given power at the wheel.

RULE.

As the diameter of the axle : is to the diameter of the wheel :: so is the power applied to the wheel : to the weight suspended to the axle.

EXAMPLE.

1. It is required to make a windlass in such a manner, that 1 lb. applied to the wheel, shall be equal to 12 lb. suspended to the axle; now allowing the axle to be 4 inches diameter, what must be the diameter of the wheel?

lb. in. lb. in.
As 1 : 4 :: 12 : 48=4 feet the diameter of the wheel. *Ans.*

2. Suppose the diameter of an axle to be 6 inches, and that of the wheel 5 feet; what power at the wheel will balance 10 lb. at the axle? *Ans.* 1 lb.

3d. OF THE SCREW.

In the screw there are four things to be considered: *viz.* the power, the weight, the distance between the threads, and the circumference. To find any one of these, the other three being given, observe the following proportional

RULE.

As the distance between the threads of the screw:
Is to the circumference ::
So is the power:
To the weight.

Note. 1. To find the circumference of the circle described by the end of the lever; multiply the double of the lever by 3,14159, and the product will be the circumference.

2 *It is usual to* abate ⅓ of the effect of the machine for the friction

EXAMPLE.

There is a screw whose threads are an inch asun
lever by which it is turned is 36 inches long, and th
to be raised a ton, or 2240 lb.; what power or fo
be applied to the end of the lever sufficient to turn t
that is to raise this weight?

Thus, the lever $36 \times 2 = 72$, and $72 \times 3{,}14159 = 2$
the circumference.

Circum. *in.* *lb.* *lb.*
Then, as 226,194 : 1 :: 2240 : 9,903 the pow

A COLLECTION OF PROMISCUOUS QUESTIONS, TO E THE SCHOLAR ON THE FOREGOING RULES

1. What is the sum of 2578, added to itself?
An.

2. What is the difference between 14676, and t
of itself? *Ans.*

3. There is the sum of 1468 dollars in three b
first contains 461, the second 581, how many a
third bag? *A*

4. What is the sum of the third and half third
lar? *Ans.*

5. What number is that which being multiplied l
product will be 1080?

6. Required the quoticnt of the square of 476
by the half of itself, or its single power? *A*

7. A general drawing up his army into a soli
found he had 231 over and above, but increasing
with one soldier, he wanted 44 to complete the squ
many men did his army consist of? *Ans.*

8. What number added to the cube of 21, will
sum equal to 113 times 147? *An*

9. A person possessed of $\frac{3}{5}$ of a ship, sold $\frac{2}{3}$ of
for 1260 dollars; what was the value of the whol
the same rate? *Ans.* 50

10. A guardian paid his ward 3500 dollars for
rs, which he had in his hands for 8 years; wha
terest did he allow him? *Ans.* 5

11. A young man received 210 dollars, which was $\frac{3}{5}$ of his elder brother's portion; now three times the elder brother's portion was half of the father's estate; how much was the estate worth? *Ans.* 1890 dolls.

12. A broker bought for his principal in the year 1720, the sum of 400 dollars capital stock, in the south sea, at 650 per cent. and sold it again when it was worth but 130 dollars per cent.; how much was lost upon the whole?

Ans. 2080 dolls.

13. A gentleman went to sea at 17 years of age; 8 years after he had a son born, who lived 46 years, and died before his father; after whom the father lived twice 20 years, and then died also; I demand the age of the father when he died? *Ans.* 111 years.

14. A, B, and C, entered into partnership in trade, A put in a sum unknown, B put in 20 pieces of cloth, and C put in 500 dollars; at the end of one year they had gained 1000 dollars, whereof A received 350 dollars for his share, and B 400 dollars; required C's share, how much A put in, and the value of B's cloth?

Ans. C's share 250 dollars,—A put in 700 dollars,
—B's cloth was worth 800 dollars.

15. A captain and 160 sailors took a prize worth 2720 dollars, of which the captain gets $\frac{1}{5}$ for his share, and the rest is equally divided among the sailors; what was each one's part?

Ans. The captain gets 544 dollars, and each sailor 13 dollars 60 cents.

16. A lady tells her husband, upon her marriage, that her fortune, the interest of which for one year at 6 per cent. was 972 dollars, was but the $\frac{3}{5}$ of the interest of her father's estate for three years, at the same rate per cent.; what was the lady's fortune, and what was the value of her father's estate?

Ans. Her fortune was 16,200 dollars, and her father' estate was 150,000 dollars.

17. A stone measures 4 feet 6 inches long, 2 feet 9 inches broad, and 3 feet 4 inches deep; how many cubic feet does it contain? *Ans.* 41 feet 3 inches.

18. Suppose $\frac{1}{5}$ of a mast or pole stands in the ground, 12 feet in the water, and $\frac{5}{8}$ of its length above the water; what is its whole length? Ans. 216 feet.

[illegible] answered [illegible] [illegible] what was his age?

Ans. [illegible] years.

[illegible] the same rate?

Ans. [illegible] dollars.

[illegible] the squares [illegible] 1890.

[illegible] from the same [illegible] each of them travels [illegible] the time in which they will have [illegible] *Ans.* [illegible] days [illegible] hours 52¼ mi.

[illegible] discharged twice with 6 lb. of [illegible] will 7 Cwt. 3qr. 17*lb.* discharge the [illegible] *Ans.* 295 times.

[illegible] which if you add ⅔ of itself, [illegible] *Ans.* 12.

[illegible] which being divided by ¾, the [illegible] 21? *Ans.* 15¾.

[illegible] which being multiplied by 15, the product will be [illegible]? *Ans.* $\frac{1}{25}$

27. What number is that, from which if you take ⅗, the remainder will be [illegible]? *Ans.* $\frac{23}{44}$

28. A gentleman wishing to distribute some money among a number of children, found he wanted 8 cents to give them 3 cents a piece, he therefore gave each 2 cents, and had three cents left; how many children were there? *Ans.* 11.

29. In what time will 500 dollars amount to 1000, at 6 per cent. per annum? *Ans.* 16 years 8 months.

30. When ½ of the members of congress were assembled in 15, there were ⅓ + 10 absent; how many members were in all? *Ans.* 150.

31. If the earth be 360 degrees round, each 69½ miles, how long would it take a man to travel once round, at 20 miles a day, admitting there were no obstacles in the way, [illegible] ing 365¼ days in the year.

Ans. 3 years 155¼ days.

32. What is the mean time for paying 100 dollars at $3\frac{1}{4}$ months, 150 dollars at $4\frac{1}{2}$ months, and 204 dollars at $5\frac{3}{4}$ months? *Ans.* 4 months $23\frac{3}{23}\frac{4}{7}$ days.

33. If A can do a piece of work alone in 7 days, and B do the same in 12, how long will it require them both together? *Ans.* $4\frac{8}{19}$ days.

34. A minor of 14 years of age, had an annuity left him of 400 dollars; this sum his guardian agreed to receive yearly, and allow him compound interest at 5 per cent. thereon, till he should arrive at 21 years of age; how much must he then receive? *Ans.* 3256 dolls. 80+cents.

35. Sold goods to the amount of 700 dollars for four months; what was the present worth, at 5 per cent. simple interest? *Ans.* 688 dolls. 52+cents.

36. Three persons, A, B, and C, purchased a lot in partnership, for which A advanced $\frac{3}{8}$, B $\frac{3}{7}$, and C 140 dollars; what sum did A and B pay, and what part of the lot belonged to C?

Ans. { A paid 267 dolls. 27+cts.
B paid 305 — $45\frac{1}{2}$ —
and C had $\frac{11}{56}$ parts.

37. A gentleman finding several beggars at his door, gave to each four cents, and had sixteen left; but if he had given to each six cents, he would have wanted 12; how many beggars were there? *Ans.* 14.

38. B and C can build a wall in 18 days, but with the assistance of A they can do it in 11 days; in what time can A do it alone?

Suppose the work to consist of 198 parts.

Then $198 \div 18 = 11$ parts performed by B and C, in one day.

Again, $198 \div 11 = 18$, performed by A, B, and C, in one day.

But $18 - 11 = 7$ parts performed by A alone.

	P.	*D.*		*P.*	*D.*	*h.*	*m.*	
And as	7	: 1	::	198	: 28	3	$25\frac{5}{7}$	*Ans.*

39. Twenty members of congress, 30 merchants, 24 lawyers, and 24 citizens, spent at a dinner 192 dollars; which sum was divided among them in such a manner, that 4 members of congress paid as much as 5 merchants, 10 merchants as much as 16 lawyers, and 8 lawyers as much

as 12 citizens; the question is to know the sum of money paid by all the members of congress; also, by the merchants, lawyers, and citizens?

Ans. The 20 members of congress paid 60 dollars the 30 merchants paid 72, the 24 lawyers paid 36 and the 24 citizens paid 24.

40. What difference is there between a piece of ground 28 perches long, by 20 broad, and two others each of half those dimensions? *Ans.* 1 acre 3 qrs.

41. Required the dimensions of a parallelogram, containing 200 acres, which is 40 perches longer than wide?

Ans. 200 perches by 160.

42. How many acres are contained in a square field, the diagonal of which is 20 perches more than either of its sides? *Ans.* 14 acres 2 qrs. 11 per.

43. The paving of a triangular yard, at 18*d.* per foot, came to 100*l.*; the longest of the three sides was 88 feet; what then was the sum of the other two equal sides?

Ans. 106,85 feet.

44. Required the length of a line by which a circle that shall contain just half an acre may be laid off?

Ans. $27\frac{3}{4}$ yards.

45. A ceiling contains 114 yards 6 feet of plastering, and the room is 28 feet broad; what is its length?

Ans. $36\frac{6}{7}$ feet.

46. A common joist is 7 inches deep, and $2\frac{1}{2}$ thick, but I want another just as big again, that shall be three inches thick; what must be its other dimensions?

Ans. $11\frac{3}{5}$ inches.

47. If 20 feet of iron railing weigh half a ton, when the bars are an inch and a quarter square, what will 50 feet come to at $3\frac{1}{2}d.$ per pound, the bars being but $\frac{7}{8}$ of an inch square? *Ans.* 20*l.* 0*s.* 2*d.*

48. A may-pole whose top being broke off by a blast of wind, struck the ground at 15 feet distance from the foot of the pole; what was its whole height, supposing the length of the broken piece to be 39 feet? *Ans.* 75 ft.

49. Required a number, from which if 7 be subtracted, and the remainder be divided by 8, and the quotient be multiplied by 5, and 4 added to the product, the square root of the sum extracted, and three-fourths of that root cubed, the cube divided by 9, the last quotient will be 24? *Ans.* 103.

50. A vintner has a cask of wine containing 500 galls. of which he draws 50 galls. and fills it up with water, and repeats the same thing five times; I demand what quantity of wine, and also of water, is then in the cask?

Ans. 295 galls. 1 qt. of wine, and 204 galls. 3 qts. of water nearly.

51. Since a pile of wood 4 feet long, 4 feet high, and 8 feet broad, makes a cord, what part of a cord will be in a pile of half the dimensions each way? *Ans.* $\frac{1}{8}$ part.

The answers to the following questions are designedly omitted, that the scholar may be induced to apply to the resources of his own mind alone for the solution thereof. Without habits of reflection and investigation are acquired, by which he can compare, examine and apply the various rules and directions that are contained in this treatise, he never can have any good claim to be considered a proficient in arithmetic

52. A owed B 1864 dollars, for which he gave his note, on interest, bearing date April 1st, 1817.

On the back of the note are the following endorsements, *viz.*

Oct. 15th, 1817. Received in cash 225 dolls. 50 cts.

Jan. 10th, 1818. Received in cash 150 —

Same date, one bag of coffee; weight 1*Cwt.* 22*lb.* at 29 cents per pound.

May 16th. Received 3 ton of iron at 195 dolls. per ton.

What is the sum due from A to B, on the 1st of August, 1818?

53. How many cords are there in a pile of wood 36 feet long, $6\frac{1}{2}$ feet wide, and $8\frac{3}{4}$ feet high?

54. If a man spends 356 dollars 34 cents per year, how much will it be per day?

55. A bankrupt, whose whole property is worth 2564 dollars $95\frac{1}{2}$ cents, can pay his creditors but $18\frac{3}{4}$ cents on a dollar; how much does he owe?

56. If 8 men spend 20 dollars 50 cents in 30 days, how long will 64 men be in spending 100 dollars at the same rate?

57. A bridge built over a stream in 6 months by 34 men, being washed away by a flood, how long time will it take 86 men to build another in its place, of twice as much work?

58. Three gardeners, A, B, and C, having bought a piece of ground, find the profits of it to amount to 240 dollars a year; now the sum of money which they gave, was in such

proportion, that as often as A paid 5 dolls. B paid 7, and as often as B paid 4 dolls. C paid 6; how much must each man receive for his share of the profits per annum?

59. If a county tax of 7 cents and 3 mills per cent. is assessed on property, how much must that man pay, whose property is valued at 8564 dollars 20 cents?

60. Suppose a cistern having a pipe which conveys 4 gallons 2 quarts into it in an hour, and has another that lets out 2 gallons 2 quarts and 1 pint in an hour; in what time will it be filled, allowing it to contain 84½ gallons?

61. What is the length of a lane, which, being 36 feet wide, will contain just one acre of ground?

62. If 50 men consume 12 bushels of grain in 30 days, how much will 40 men consume in 90 days?

63. A gentleman had 18 dollars 90 cents to pay among his laborers; to every boy he gave 6 cents, to every woman 8 cents, and to every man 16 cents; now there were three women for every boy, and two men for every woman; required the number of each?

64. Two men depart from the same place, and travel the same way; the one travels at the rate of 3 miles an hour, for 8 hours every day; the other goes at the rate of 4½ miles, for 7 hours each day; how far are they apart at the end of 13 days?

65. A began to trade on the 1st of January, with a capital of 962 dollars; on the 15th of April following, he took in B as a partner, with 1635 dollars; on the 1st of July, A put in 320 dollars more, and 1 month after B drew out ¼ of his capital; on the last day of December, on settling their accounts, they found a gain of 486 dollars 64 cents; what was each partner's share?

66. Suppose the Ohio river to be 2500 feet wide, 6 feet deep, and runs at the rate of 3 miles an hour; in what time will it fill a cistern of two miles in length, breadth, and depth, the mile being 5280 feet?

67. A sloth was observed climbing a tree at the rate of 9½ inches every day, but during the night slipped down 6¾ inches; how long will it be in reaching a limb 45 feet 6 inches from the ground?

68. In an orchard of fruit trees, ½ of them bear apples, ⅛ peaches, ⅛ cherries, ⅛ plums, and 46 are pears; how many trees does the orchard contain?

69. *An old soldier* lately received a sum of money as

pension from government: of this sum he paid 94 dollars in the payment of debts which he then owed, half of what remained he lent to a friend, and the fifth he gave for a suit of clothes; he then found that nine-tenths of his money was gone; what sum did he at first receive?

70. What number is that, of which the difference between its third and fourth parts is 84?

71. In turning a chaise within a circle of a certain diameter, it was discovered that the outer wheel turned thrice, while the inner turned twice; now supposing the axle-tree 4 feet long, and the wheels of an equal size, the length of the circumference described by each wheel is required?

72. The sum of the sides of an equilateral triangle is 125 feet; required the area thereof?

A, in a scuffle, seized on $\frac{3}{8}$ of a parcel of sugar-plums; B catched three-eighths of it out of his hands, and C laid hold on three-tenths more, D ran off with all that A had left except one-seventh, which E afterwards secured slily for himself: then A and C jointly set upon B, who in the conflict let fall $\frac{1}{5}$ he had, which was equally picked up by D and E: B then kicked down C's hat, and to work they went anew for what it contained; of which A got $\frac{1}{4}$, B $\frac{1}{5}$, D two-sevenths, and C and E equal shares of what was left of that stock: D then struck $\frac{3}{4}$ of what A and B last acquired out of their hands; they with difficulty recoved five-eighths of it in equal shares again, but the other three carried off one-eighth apiece of the same. Upon this they called a truce, and agreed, that the $\frac{1}{5}$ of the whole, left by A at first, should be equally divided among them—How much of the prize, after this distribution, remained with each of the competitors?

Ans. A got 2863, B 6335, C 2438, D 10294, and E 4950.

Solution.

First, $\frac{3}{8}$ of $\frac{2}{3}=\frac{1}{4}$ B's } First acquisition
And $\frac{3}{10}$ of $\frac{2}{3}=\frac{1}{5}$ C's } $=\frac{9}{20}$ their sum
Then $\frac{2}{3}-\frac{9}{20}=\frac{13}{60}$, or $\frac{91}{420}$ left
$\frac{1}{7}$ of $\frac{13}{60}=\frac{13}{420}$ E's first acquisition.
Also, $\frac{91}{420}-\frac{13}{420}=\frac{78}{420}$ D's. Thus ended the 1st heat.

Again, $\frac{1}{2}$ of $\frac{1}{4}$ $=\frac{1}{8}$ B's
Retained $\frac{1}{8}$ C's } Part, at the end of the second scuffle.
And $\frac{78}{420}+\frac{1}{16}=\frac{129}{560}$ D's
Also $\frac{13}{420}+\frac{1}{16}=\frac{157}{1680}$ E's

Proceeding, $\frac{1}{4}$ of $\frac{1}{5}$ $=\frac{1}{20}$ A's
$\frac{1}{2}$ of $\frac{1}{5}=\frac{1}{15}+\frac{1}{8}=\frac{23}{120}$ B's } Their situation at the end of the 3d attack.
$\frac{2}{7}$ of $\frac{1}{5}=\frac{2}{35}+\frac{139}{560}=\frac{171}{560}$ D's
Then $\frac{1}{20}+\frac{1}{15}+\frac{2}{35}=\frac{73}{420}$ to be taken from C's. Thus, $\frac{1}{5}-\frac{73}{420}=\frac{11}{420}$
and $\frac{1}{2}$ of $\frac{11}{420}$ $=\frac{11}{840}$ C's
And $\frac{157}{1680}+\frac{11}{340}$ $=\frac{119}{1680}$ E's

A. B.

Further, $\frac{1}{20}+\frac{1}{15}=\frac{7}{60}$, and $\frac{3}{4}$ of $\frac{7}{60}=\frac{7}{80}$ lost by A and B.

Then, $\frac{5}{16}$ of $\frac{7}{80}+\frac{1}{4}$ of $\frac{1}{20}$ $=\frac{51}{1280}$ A's
Also, $\frac{5}{16}$ of $\frac{7}{80}+\frac{1}{4}$ of $\frac{1}{15}+\frac{1}{8}$ $=\frac{649}{3840}$ B's
And $\frac{1}{8}$ of $\frac{7}{80}+\frac{11}{840}$ $=\frac{323}{13440}$ C's
$\frac{1}{8}$ of $\frac{7}{60}+\frac{171}{660}$ $=\frac{1417}{4480}$ D's
$\frac{1}{8}$ of $\frac{7}{80}+\frac{179}{1680}$ $=\frac{1579}{13440}$ E's

} Part, after the last heat and before the truce.

$\frac{1}{5}$ of $\frac{1}{3}$ $=\frac{1}{15}$

Then, $\frac{51}{1280}+\frac{1}{15}=\frac{2863}{26880}$ A's
$\frac{649}{3840}+\frac{1}{15}=\frac{6335}{26880}$ B's
$\frac{323}{13440}+\frac{1}{15}=\frac{2438}{26880}$ C's
$\frac{1417}{4480}+\frac{1}{15}=\frac{10294}{26880}$ D's
$\frac{1579}{13440}+\frac{1}{15}=\frac{4950}{26880}$ E's

} Share, carried off at last.

So that if the sugar-plums were 26880, then

A got	2863
B	6335
C	2438
D	10294
E	4950

Ans.

QUESTIONS FOR EXAMINATION.

THIS collection of questions is designed to assist the teacher in the examination of his scholars. It will contribute very much to the progress of scholars, to assign them a certain number of these questions as lessons, to be answered correctly and with facility. Many similar questions will no doubt, from time to time, occur to the mind of the teacher, on the different sections, as the scholar proceeds. By accustoming his pupils to answer such with ease, not only will his own burden in teaching be lessened, but the parents of children, who have been intrusted to his care, will find that neither their trouble or expense has been in vain.

PART I.

What is Arithmetic? How many parts does it consist of?
What are the characters used in arithmetic?
What is numeration? How are the digits divided?
What is the rule for writing numbers? What is simple addition?
How do you place numbers to be added? How is the sum or amount of each column to be set down? Why do you carry at 10, rather than for any other number? How is addition proved? What is simple subtraction? How must the given numbers be placed?
How is subtraction performed? How is subtraction proved?
What is simple multiplication? What are the numbers called?
In what order are the numbers in multiplication to be placed?
How many cases are there in multiplication? How is the operation to be *performed in the first* and second cases? How is multiplication proved?
When there are ciphers on the right-hand of either of the factors, *how do you proceed?* What is simple division?
What are the given numbers called? How are they to be placed?

How many cases are there in division?
How is division performed in each case? When the multiplier is the exact product of any two factors, how do you proceed?
When there are ciphers on the right of the divisor, how do you proceed?
When the divisor is 10, 100, 1000, &c. how do you proceed?
How is division proved?

PART II.

Federal money, why so called?
What are its denominations, and standard weights?
How is addition, subtraction, multiplication, and division severally performed in federal money? What is compound addition?
How is compound addition performed? How is it proved?
What are the denominations of English money, and how are they valued?
What articles is troy weight used for? What are its denominations and how valued? What articles is avoirdupois weight used for?
What are its denominations, and how valued? What is apothecaries' weight used for? What are its denominations and how valued?
What are the denominations of cloth measure, and how valued?
What are the denominations of long measure, and how estimated?
What are the denominations of land measure, and how rated?
What is cubic measure, what are its denominations, and relative difference?
What are the denominations of time, and what their relative differences?
What is the exact length of the solar year?
What are the denominations of motion, and the relative difference?
For what is liquid measure used, what its denominations, and relative difference? What are the denominations of dry measure, what used for and how estimated?
What is compound subtraction, and how performed?
What is compound multiplication, how many cases, and how performed?
What is compound division, how many cases, and how performed?
What is reduction, and how performed? How is reduction proved?
How are pence reduced to cents, Penn. currency? How are pounds, shillings, and pence, reduced to dollars, Pennsylvania currency?

PART III.

What is decimal arithmetic, and how distinguished from whole numbers?
What is the decimal point called?
What effect has ciphers, placed on the right-hand of the integer, and what effect when placed on the left-hand? How is addition of decimals performed? How is subtraction of decimals performed?
How is multiplication of decimals performed? How is division of decimals performed? How many cases in reduction of decimals?
How is a vulgar fraction reduced to a decimal?
How are numbers of different denominations reduced to a decimal of equal value? How are decimals reduced to their equal value in integers?

PART IV.

What is proportion? Into how many parts is it divided?
What are the given terms in proportion called?
What is required in the single rule of three direct?
How may you know when the question is in direct proportion?
What is the rule for stating questions in the single rule of three direct?
How is the operation performed? How do you prove questions in the single rule of three direct? What is the single rule of three inverse?
How may you know when the question is in the single rule of three inverse?
How is the operation performed in the single rule of three inverse? How are questions proved in this rule? What is the double rule of three?
How many, and which terms must be a supposition, and how many, and *which must be a* demand?
What is the rule for stating questions in the double rule of three?
How is the operation performed in the double rule of three direct?

How do you know when the question is in direct proportion, and when in inverse? How is the operation performed in inverse proportion?

PART V.

What is practice, and why so called? How many cases are there in practice?
When the price consists of dollars, cents, and mills, how is the operation performed? When the price is the fractional part of a dollar or cent, how is the operation performed? When the price and quantity given are of several denominations, how is the operation performed?
When the price consists of pounds, shillings, pence, and farthings, how do you proceed? What is meant by aliquot parts?
When both the price of the integer and the quantity are of different denominations, how do you proceed?
What is tare and tret, and what is gross and neat?
How do you work questions in tare and tret? What is interest?
What is the general rate of interest? What is the sum of money loaned, called? What do you understand by the amount?
How many kinds of interest are there? What is simple interest?
How many cases are there in simple interest? When the given time is years and the principal dollars, how is the interest found?
When there are cents and mills in the principal, how do you proceed?
When the time is years and months, or months only, how is the interest found? When the time is months and days, or days only, how is the operation performed? How is the interest computed on bonds, notes, &c.?
What is compound interest, and how is it performed? What is insurance?
What is the instrument of agreement termed?
How are the questions in insurance performed?
What is commission, and how performed? What is brokage, and how performed? What is stock, and how bought and sold?
What is rebate or discount, and what the rule to work questions therein?
What is the difference between discount and interest?
What is bank discount, and how is the discount calculated?
What do you mean by the equation of payments?
How is the mean time found in the equation of payments?
What is fellowship, and how many kinds are there?
What is single fellowship, and how is the operation performed?
What is compound fellowship, and what is the rule for working questions therein? What is profit and loss, and what is the rule of operation therein? What is barter, and how performed?
What is exchange, and of how many kinds?
What do you understand by *par* in exchange, and what by *agio*?
How do you reduce the currency of different states to federal money?
How do you reduce the currency of one state to another, where it is different in them? How are accounts kept in England, Ireland, and how in France, Spain, &c.? What is alligation, and how many cases are therein?
How are the operations performed in the first case, second case, &c.?

PART VI.

What is a vulgar fraction, and how many kinds are there?
What is a proper fraction, what is an improper fraction, what is a compound fraction, and what is a mixed fraction?
What are the numbers above the line called, and also those below?
How do you reduce vulgar fractions to their lowest terms?
How are mixed numbers reduced to an improper fraction?
How is an improper fraction reduced to a whole or mixed number?
How do you reduce fractions to others that shall have a common denominator? How do you find the least common denominator?
How do you find the value of a fraction, in the known parts of an integer?
How are given quantities reduced to the fraction of a greater denomination?
How are vulgar fractions reduced to decimals of the same value?
How do you reduce a compound fraction to a single one?
How are vulgar fractions added, subtracted, multiplied, and divided?

How do you perform the single rule of three in vulgar fractions, direct, and inverse?

PART VII.

What is involution? What do you understand by the power of a number?
How is involution performed? What is the number denoting the power, termed? What is evolution? What do you understand by a root?
How do you extract the square root?
If there be decimals in the given number, how must it be pointed?
How do you extract the square root of a vulgar fraction?
How is the square root of a mixed number extracted?
How do you find the side of a right angled triangle, the other two being given? How do you find the side of a square, in any given area?
How do you find the diameter of a circle, when the area is given?
How do you prove the square root? What is a cube?
How do you extract the cube root in whole numbers?
How do you extract the cube of a vulgar fraction?
How do you point off in decimal numbers? How is the cube root of a mixed number extracted? How is the cube root proved?
What is progression, and how many kinds? What is principally to be observed in arithmetical progression? How do you find the last term; and sum of all the terms? How do you find the common difference?
What is geometrical progression, and how does it differ from arithmetical?
What is principally to be observed in geometrical progression?
How do you find the last term, and sum of all the series?
What is position, and of how many kinds? How do you resolve questions in single position? How is single position proved? What is double position? What is the rule for working questions in double position?
What do you understand by permutation? How is the number of variations found in this rule? What do we learn from the results of this rule?
What is combination? How do you find the greatest possible number of combinations in any given number?

PART VIII.

What are duodecimals? What are the denominations in duodecimals, and what are they termed? How are duodecimals added, subtracted, and multiplied? How do you prove multiplication of duodecimals?
How is the solid content of bales, &c. found by duodecimals?
How do you find a ship's tonnage? What is the carpenters' rule, and what its use? How do you find the superficial content of boards, &c.?
How do you find the solid content of squared timber?
How do you find the solid content of round timber?
What things belong to carpenters' work? By what numbers do carpenters usually measure their work? How is brick-work estimated?
What is the standard thickness of a brick wall?
How do you reduce a wall of a different thickness to a standard one?
How is masons' work measured? What kind of measure is used for materials? How is the solid content of walls calculated?
What is superficial measure? How is plasterers' work divided?
In what manner is plasterers' work measured? How is white-washing and coloring estimated? How is pavers' work calculated?
How do painters compute their work? In what manner is glaziers' work estimated? How are the contents of squares calculated?
In what way is the area of an oblong piece of ground ascertained?
How do you calculate triangular pieces of ground?
How do you calculate a piece of ground lying in the shape of an oblique parallelogram? How are pieces of ground, bounded by four irregular sides, calculated? How do you calculate the area of a circle?
What is gauging? How are the contents of casks, &c. calculated?
How do you calculate the power of the lever?—of the axle and wheel?
What things are to be considered in finding the power of the screw?
How do you find the proportion between the weight and the power?

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